Estimating Optimal Stopping Rules in the Multiple Best Choice Problem with Minimal Summarized Rank via the Cross-Entropy Method

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Abstract. The best choice problem is an important class of the theory of optimal stopping rules. In this article, we present the Cross-Entropy method for solving the multiple best choice problem with the minimal expected ranks of selected objects. We also compare computation results by Cross-Entropy method with results by the genetic algorithm. Computational results showed that the Cross-Entropy method is producing high-quality solution.

1 Introduction

The best choice problem is an important class of the theory of optimal stopping rules. It has been studied by many authors: Chow, Robbins, and Siegmund [2], Dynkin and Yushkevich [4], Gilbert and Mosteller [6], Shiryaev [13].

In this chapter we consider the multiple best choice problem [9], [10]. We have a known number $N$ of objects numbered 1, 2, ..., $N$, so that, say, an object numbered 1 is classified as "the best", ..., and an object numbered $N$ is classified as "the worst". It is assumed that the objects arrive one by one in random order, i.e all $N!$ permutations are equiprobable. It is clear from comparing any two of these objects which one is better, although their actual number still remain unknown. After having known each sequential object, we either accept this object (and then a choice of one object is made), or reject it and continue observation (it is impossible to return to the rejected object). The object is to find a stopping rule which minimizes the expected absolute rank of the individual selected.

We can use this model to analyse some behavioral ecology problems such as sequential mate choice or optimal choice of the place of foraging. Indeed, in some species, active individuals (generally, females) sequentially mate with different passive individuals (usually males) within a single mating period (see, e.g., Gabor and

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Note also that an individual can sequentially choose more than one place to forage. So we can consider the random variable as quality of item (potential mate or place of foraging) which appears at time $t$.

For one choice problem the minimal expected rank for the rank minimization problem tends to the value $\prod_{j=1}^{\infty} (1 + \frac{2}{j})^{\frac{1}{1-j}} \approx 3.8695$ \[6\]. The model considered in \[7\] is similar to the one choice problem. But instead of being a fixed integer $N$, the total number of individuals is a strictly positive, integer-valued, bounded random variable. The author studied the asymptotic behavior of the minimal expected rank. Bruss and Ferguson \[1\] considered this problem in full information setting where the decision is based on the actual values associated with the applicants, assumed to be independent and identically distributed from a known distribution. Tamaki considered the best choice problem which allows the applicant to refuse an offer of acceptance with probability $1 - p$, $0 < p < 1$ \[15\].

The rest of the chapter is organized as follows. Section 2 introduces the multiple best problem with minimal summarized rank. In Section 3 we describe the cross-entropy method. Section 4 discusses cross-entropy method for the multiple best problem with minimal summarized rank. Section 5 presents the experiment results of the cross-entropy method for the problem. In Section 6 we explain the genetic algorithm. Section 7 discusses numeric results of the genetic algorithm for the problem. Finally, the conclusions are given.

## 2 The Multiple Best Choice Problem with Minimal Summarized Rank

Let we have $N$ objects, which are ordered on quality. At time $n$ we can compare current object with all previous objects, but we nothing know about quality remaining $N - n$ objects. After getting acquainted with $a_n$ we can it or accept (and then the choice of one object is made), or reject and continue observation (we can’t return to rejected object).

Let $x_i$ be an absolute rank of selected object, i.e. $x_i = 1 + \text{number of objects from } (a_1, a_2, \ldots, a_N) < a_i$. The objective is to find optimal procedure such that the expected gain $E(x_{\tau_1} + \ldots + x_{\tau_k})$, $k \geq 2$ is minimal.

Denote by $(a_1, a_2, \ldots, a_N)$ any permutation of numbers $(1, 2, \ldots, N)$, 1 corresponds to the best object, $N$ corresponds to the worst one. All $N!$ permutations being equally likely. For any $i = 1, 2, \ldots, N$ let $y_i = \text{number of terms } a_1, a_2, \ldots, a_i$ which are $\leq a_i$, and $y_i$ is called the relative rank of the $i$th object. As $y_1, y_2, \ldots, y_N$ are independent, and

\[
P(y_i = j) = \frac{1}{i} \quad (j = 1, 2, \ldots, i),
\]

\[
P(x_i = k \mid y_1 = l_1, \ldots, y_{i-1} = l_{i-1}, y_i = j) = P(x_i = k \mid y_i = j)
\]

\[
= \frac{\binom{l_{i-1}}{k-1} \cdot \binom{N-j}{N-k}}{\binom{N}{N}}.
\]