The Computational Complexity of RaceTrack

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Abstract. Martin Gardner in the early 1970's described the game of RaceTrack [M. Gardner, Mathematical games—Sim, Chomp and Race Track: new games for the intellect (and not for Lady Luck), *Scientific American*, 228(1):108–115, Jan. 1973]. Here we study the complexity of deciding whether a RaceTrack player has a winning strategy. We first prove that the complexity of RaceTrack reachability, i.e., whether the finish line can be reached or not, crucially depends on whether the car can touch the edge of the carriageway (racetrack): the non-touching variant is NL-complete while the touching variant is equivalent to the undirected grid graph reachability problem, a problem in L but not known to be L-hard. Then we show that single-player RaceTrack is NL-complete, regardless of whether driving on the track boundary is allowed or not, and that deciding the existence of a winning strategy in Gardner's original two-player game is P-complete. Hence RaceTrack is an example of a game that is interesting to play despite the fact that deciding the existence of a winning strategy is most likely not NP-hard.

1 Introduction

RaceTrack is a popular multi-player simulation pencil-paper game of car racing. The origin of the game is not clear, but most people remember this game from high school, where great effort and time was spent to become the RaceTrack champion, even at the expense of the said champion's school results. Variants of the game appeared all over the world under various names like, e.g., *Le Zip* in France or *Vektorrennen* in Germany. Here is the game description, literally taken from Gardner [4]: The game is played on math-paper where a racetrack is drawn. Then the cars are lined up at a grid position at the start line, and at each turn a player moves his car along the track to a new grid position subject to the following rules:

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1. The new grid point and the straight line segment joining it to the preceding grid point must lie entirely within the track.
2. No two cars may simultaneously occupy the same grid point, i.e., no collisions are allowed.
3. Acceleration and deceleration are simulated as follows: a car maintains its speed in either direction or it can change its speed by only one distance unit per move—see Figure 1 for illustration. The first move following this rule is one unit horizontally or vertically, or both.

The first car to cross the finish line (point) wins. A car that collides with another car or leaves the track is out of the race.

Here we investigate the complexity of RaceTrack when played as a 1-player or 2-player game. Before describing our results, we note that the shape of the racetrack border is *a priori* arbitrary. Thus, in some far-fetched settings, merely verifying whether a move is valid, i.e., merely checking whether the new grid point and the straight line segment joining it to the preceding grid point lies entirely within the track, could be undecidable. To avoid such complications, we stick to a discrete version of the racetrack border, where the track is drawn along grid lines. This is a reasonable restriction, which does not change the practical appeal of the game. Figure 1 shows the discretization of an arbitrarily shaped track.

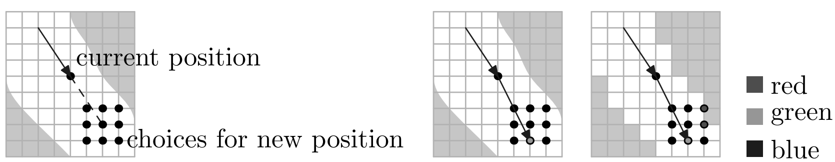

Fig. 1. (Left:) The arrow depicts the last move of the car—two squares east and three squares south—on the racetrack (drawn in white); the gray shaded area is outside of the racetrack. If the car maintains its speed it will follow the dashed line and go two squares east and three squares south again, but it can also reach one extra square north, south, east, or west of this point by changing speed. These extra points are marked by black dots. (Middle:) One out of nine legal moves are shown. (Right:) Discrete version of the RaceTrack game. Observe that certain movements of the car are not possible anymore if the border is *not* allowed for driving; here 7 out of 9 movements remain.

Solitaire RaceTrack will refer to the problem of deciding whether a single player can reach the finish line within an input-specified number of legal moves. By RaceTrack *reachability*, we will mean the simpler problem of deciding whether the single player can reach the finish line at all. In the first part of this paper, we reduce the *touching* variant of RaceTrack reachability (i.e., with the track border considered part of the driving area) to the undirected grid graph reachability problem, and deduce from [1] that this touching variant is NC^1-hard and can be solved in deterministic logarithmic space. By contrast, and to our