At the end of the 1960s, René Thom was the first scientist to develop a general mathematical theory of morphogenetic processes. This chapter presents the fundamental principles of that theory.

### 15.1 General Content of the Model

Let $S$ be a system satisfying the following hypotheses:

a. There exists an internal process $X$ which defines the internal states that system $S$ can occupy in a stable manner, and the number of these states is finite;

b. the internal states of $S$ are in competition with each other and mutually determine each other, and the choice of one of them as the actual state makes the others virtual;

c. there therefore exists an instance of selection $I$ which, on the basis of criteria specific to the system, selects the actual state from among the possible internal states;

d. the system $S$ is controlled by a certain number of control parameters varying within a space $W$ which we call, to distinguish it from the internal process $X$, the external space (or control space or substrate space) of $S$.

We also assume that the control is continuous, in the sense that the internal process $X$ is a process $X_w$ which depends continuously on the value $w$ of the control. This process varies when $w$ varies in $W$ and when it is deformed it also deforms the structure of the internal states and their relations of mutual determination. We denote $\mathcal{X}$ the space of possible internal processes $X$. If the above hypotheses are verified, the
system $S$ will be described firstly by the (continuous) field $\sigma : W \to \mathcal{X}$ associating $w \in W$ with the process $X_w$, and then by the instance of selection $I$.

The standard example is that of the thermodynamic phenomena of phase transitions. In this case, the system $S$ is the thermodynamic system considered, the internal states are the thermodynamic phases (solid, liquid, gas), the instance of selection $I$ is provided by the principle of free energy minimisation and the control parameters are, for example, pressure and temperature. As for the internal process $X_w$, indescribable because of its complexity, it is the process of molecular dynamics. When the control parameters cross certain specific values, known as critical values,\(^1\) they present phase transitions, i.e. discontinuities in their observable qualities and abrupt transformations of their internal state. The critical values constitute a subset $K$ of $W$, determining the phase diagram: $K$ partitions $W$ into domains corresponding to the different phases that $S$ can present. In other words, it categorises it and externalises therein the competition between internal states, in the form of a system of discontinuities.

This is a direct consequence of hypotheses (a)–(d). A system $S = (W, \mathcal{X}, \sigma, I)$ manifests itself phenomenologically by the observable qualities $q^1, \ldots, q^n$ expressing its internal state. In other words, the internal process $X_w$ is externalized in perceptible qualities $q_w^i$. When the control $w$ varies continuously, the actual internal state varies continuously (hypothesis (d)) and the qualities $q_w^i$ therefore vary equally. But phenomenologically speaking, a continuous variation is no more than a form of qualitative invariance. It is therefore not significant. So René Thom denoted by regular point of $W$ a value $w$ of the control such that the observable qualities $q_w^i$ vary continuously – and therefore remain stable – throughout a neighbourhood $U$ of $w$ (this obviously presupposes that we have defined the concept of neighbourhood on $W$, i.e. a topology). By definition, the regular points constitute an open set $R_W$ of $W$, the open set of quality stability. Then let $K_W$ be the closed set complementary to $R_W$ in $W$. By definition, the points of $K_W$ are the values $w$ of the control such that at least one observable quality $q_w^i$ suffers a discontinuity. These are critical values, crossing which the system $S$ presents critical behaviour. They are also called catastrophic values, the closed set $K_W$ being called the catastrophic set of $S$. The $K_W$ are also called external morphologies.

As René Thom often stressed, this concept of morphology is purely phenomenological. But it is closely connected to the mathematical concept of bifurcation. Let us suppose that the control $w$ follows a path $\gamma$ in $W$. Let $A_w$ be the actual internal state initially selected by $I$. During the deformation of $X_w$ along $\gamma$ – and therefore, under hypothesis (d), of the structure of $A_w$ and under hypothesis (b), of the relations of mutual determination it has with the virtual states $B_w, C_w, \text{etc.}$ – when $A_w$ crosses a critical value, it no longer satisfies the criteria of selection imposed by $I$ under

\[^1\] Here, the term critical value is related to bifurcation theory (and, in the rest of this chapter, to catastrophe theory), and not to the language of thermodynamics. As a general rule, these critical values do not correspond to a critical point in the thermodynamic sense (a particular point where the distinction between the different phases disappears and the phase transition becomes continuous, the singularity manifesting itself in thermodynamic derivatives, of free energy, for example).