New Lower Bounds for the Vehicle Routing Problem with Simultaneous Pickup and Delivery

Anand Subramanian\(^1\), Eduardo Uchoa\(^2\), and Luiz Satoru Ochi\(^1\)

\(^1\) Universidade Federal Fluminense, Instituto de Computação, Rua Passo da Pátria 156 - Bloco E - 3\(^{\circ}\) andar, Niterói-RJ 24210-240, Brazil
\(^2\) Universidade Federal Fluminense, Departamento de Engenharia de Produção, Rua Passo da Pátria 156 - Bloco E - 4\(^{\circ}\) andar, Niterói-RJ 24210-240, Brazil

Abstract. This work deals with the Vehicle Routing Problem with Simultaneous Pickup and Delivery. We propose undirected and directed two-commodity flow formulations, which are based on the one developed by Baldacci, Hadjiconstantinou and Mingozzi for the Capacitated Vehicle Routing Problem. These new formulations are theoretically compared with the one-commodity flow formulation proposed by Dell’Amico, Righini and Salani. The three formulations were tested within a branch-and-cut scheme and their practical performance was measured in well-known benchmark problems. The undirected two-commodity flow formulation obtained consistently better results. Several optimal solutions to open problems with up to 100 customers and new improved lower bounds for instances with up to 200 customers were found.

Keywords: Vehicle Routing, Simultaneous Pickup and Delivery, Commodity Flow Formulations.

1 Introduction

The Vehicle Routing Problem with Simultaneous Pickup and Delivery (VRP-SPD) is a variant of the Capacitated Vehicle Routing Problem (CVRP), in which clients require both pickup and delivery services. This problem was first proposed two decades ago by Min [1]. The VRPSPD is clearly \(\mathcal{NP}\)-hard since it can be reduced to the CVRP when all the pickup demands are equal to zero. Practical applications arise especially in the Reverse Logistics context. Companies are increasingly faced with the task of managing the reverse flow of finished goods or raw-materials. Thus, one should consider not only the Distribution Logistics, but also the management of the reverse flow.

The VRPSPD can be defined as follows. Let \(G = (V, E)\) be a complete graph with a set of vertices \(V = \{0, \ldots, n\}\), where the vertex 0 represents the depot and the remaining ones the customers. Each edge \(\{i, j\} \in E\) has a non-negative cost \(c_{ij}\) and each client \(i \in V' = V - \{0\}\) has non-negative demands \(d_i\) for delivery and \(p_i\) for pickup. Let \(C = \{1, \ldots, m\}\) be a set of homogeneous vehicles with capacity \(Q\). The VRPSPD consists in constructing a set up to \(m\) routes in such a way that: (i) every route starts and ends at the depot; (ii) all the pickup and
delivery demands are accomplished; (iii) the vehicle’s capacity is not exceeded in any part of a route; (iv) a customer is visited by only a single vehicle; (v) the sum of costs is minimized.

Although heuristic strategies are by far the most employed to solve the VRP-SPD, some exact algorithms were also explored in the literature. A branch-and-price algorithm was developed by Dell’Amico et al. [2], in which two different strategies were used to solve the subpricing problem: (i) exact dynamic programming and (ii) state space relaxation. The authors managed to find optimal solutions for instances with up to 40 clients. Angelelli and Manisini [3] also developed a branch-and-price approach based on the set covering formulation, but for the VRPSPD with time-windows constraints. The subproblem was formulated as a shortest-path with resource constraints but without the elementary condition and it was solved by applying a permanent labeling algorithm. The authors were able find optimal solutions for instances with up to 20 clients. Three-index formulations for the VRPSPD were proposed by Dethloff [4] and Montané and Galvão [5], however only the last authors had tested it in practice. They ran their formulation in CPLEX 9.0 within a time limit of 2 hours and had reported the lower bounds produced for benchmark instances with 50-400 customers.

In this work we propose an undirected and a directed two-commodity flow formulations for the VRPSPD. These formulations extend the one developed by Baldacci et al. [6] for the CVRP. They were compared with the one-commodity flow formulation presented by Dell’Amico et al. [2]. In addition, the three formulations were implemented within a branch-and-cut algorithm, including cuts from the CVRPSEP library [7], and they were tested in well-known benchmark problems with up to 200 customers.

The remainder of this paper is organized as follows. Section 2 describes the one-commodity flow formulation [2]. In Section 3 we present the undirected and the directed two-commodity flow formulations for the VRPSPD and we compare these formulations with the one developed in [2]. Section 4 contains the experimental results obtained by means of a branch-and-cut algorithm. Section 5 presents the concluding remarks.

2 One-Commodity Flow Formulation

Reasonably simple and effective formulations for the CVRP can be defined only over the natural edge variables (arc variables in the asymmetric case), see [8]. Similar formulations are not available for the VRPSPD. This difference between these two problems can be explained as follows. In the CVRP, the feasibility of a route can be determined by only checking whether the sum of its client demands does not exceed the vehicle’s capacities. In contrast, the feasibility of a VRPSPD route depends crucially on the sequence of visitation of the clients.

The following directed one-commodity flow formulation for the VRPSPD was proposed by Dell’Amico et al. [2]. Define $A$ as the set of arcs consisting of a pair of opposite arcs $(i, j)$ and $(j, i)$ for each edge $\{i, j\} \in E$ and let $D_{ij}$ and $P_{ij}$ be the flow variables which indicate, respectively, the delivery and pickup loads