3. Solutions to Exercises

Worked solutions are provided only for the questions which are not programming exercises.

1 Dramatis Personae

(i) For each of the schemes in 11.9(i), write out the stencils of the scheme.
Use the notation which inserts a ‘∗’ at the place of an edge-vertex.

\[ [1, 4, 6, 4, 1]/8 \text{ has stencils } [1 \ 6 \ 1]/8 \]
\[ \text{ and } [4 \ \ast \ 4]/8. \]

\[ [1, 3, 3, 1]/4 \text{ has stencils } [1 \ \ast \ 3]/4 \]
\[ \text{ and } [3 \ \ast \ 1]/4. \]

The symmetry of these two stencils means that often only one of the two is explicitly described.

\[ [1, 3, 6, 7, 6, 3, 1]/9 \text{ has stencils } [3 \ \ast \ 6]/9, \]
\[ [1 \ 7 \ 1]/9 \]
\[ \text{ and } [6 \ \ast \ 3]/9. \]

\[ [1, 3, 5, 5, 3, 1]/6 \text{ has stencils } [5 \ \ast \ 1]/6, \]
\[ [3 \ \ast \ 3]/6 \]
\[ \text{ and } [1 \ \ast \ 5]/6. \]

\[ [-1, 0, 9, 16, 9, 0, -1]/16 \text{ has stencils } [-1 \ 9 \ \ast \ 9 \ -1]/16 \]
\[ \text{ and } [0 \ 16 \ 0]/16. \]

The latter is not usually made explicit, as the scheme is interpolating.
(ii) For each of the schemes in 11.9(i), write out the part of the matrix with non-zero principal diagonal. What complications did you find in interpreting this question?

\[
\begin{bmatrix}
1 & 6 & 1 \\
4 & 4 & \\
1 & 6 & 1
\end{bmatrix}
\]

\[1, 4, 6, 4, 1]/8 \] has matrix

\[
\begin{bmatrix}
1 & 3 \\
3 & 1 \\
3 & 1
\end{bmatrix}
\]

\[1, 3, 3, 1]/4 \] has matrix

\[
\begin{bmatrix}
1 & 7 & 1 \\
6 & 3 & \\
3 & 6 & 1
\end{bmatrix}
\]

\[1, 3, 6, 7, 6, 3, 1]/9 \] has matrices

\[
\begin{bmatrix}
1 & 3 \\
1 & 5 \\
-1 & 9 & -1
\end{bmatrix}
\]

\[[-1, 0, 9, 16, 9, 0, -1]/16 \] has matrix

The expected complication is that the ternary schemes have two matrices each.

(iii) How can the denominator of a scheme be determined from the arity and the sequence of integers in the 11.9(i) examples? Equally, how can the arity be determined from that sequence and the denominator?

Because each stencil has to be a weighted mean, its entries must add up to 1. The number of stencils is equal to the arity, and so the sum of all values in the mask must equal the arity. To get the denominator, therefore divide the total of the mask numerator entries by the arity.

Similarly, if you have the denominator divide that total by it to get the arity.