Building \((1 - \epsilon)\) Dominating Sets Partition as Backbones in Wireless Sensor Networks Using Distributed Graph Coloring

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Abstract. We recently proposed in [19,20] to use sequential graph coloring as a systematic algorithmic method to build \((1 - \epsilon)\) dominating sets partition in Wireless Sensor Networks (WSN) modeled as Random Geometric Graphs (RGG). The resulting partition of the network into dominating and almost dominating sets can be used as a series of rotating backbones in a WSN to prolong the network lifetime for the benefit of various applications. Graph coloring algorithms in RGGs offer proven constant approximation guarantees on the chromatic number. In this paper, we demonstrate that by combining a local vertex ordering with the greedy color selection strategy, we can in practice, minimize the number of colors used to color an RGG within a very narrow window of the chromatic number and concurrently also obtain a domatic partition size within a competitive factor of the domatic number. We also show that the minimal number of colors results in the first \((\delta + 1)\) color classes being provably dense enough to form independent sets that are \((1 - \epsilon)\) dominating. The resulting first \((\delta + 1)\) independent sets, where \(\delta\) is the minimum degree of the graph, are shown to cover typically over 99% of the nodes (e.g. \(\epsilon < 0.01\)), with at least 20% being fully dominating. These independent sets are subsequently made connected through virtual links using localized proximity rules to constitute planar connected backbones. The novelty of this paper is that we extend our recent work in [20] into the distributed setting and present an extensive experimental evaluation of known distributed coloring algorithms to answer the \((1 - \epsilon)\) dominating sets partition problem. These algorithms are both topology and geometry-based and yield \(O(1)\) times the chromatic number. They are also shown to be inherently localized with running times in \(O(\Delta)\) where \(\Delta\) is the maximum degree of the graph.

Keywords: Domatic partition problem, \((1 - \epsilon)\) dominating sets partition, Wireless Sensor Network, Graph coloring, Distributed Algorithm.

1 Introduction

In a random dense deployment of sensor networks, we can take advantage of the redundancy and physical proximity of nodes and require that only a subset of
them stay active at one time to fulfill the application’s objectives (e.g. coverage, data gathering, monitoring) while the rest of the nodes stay in a sleep mode to conserve their energy [27]. For this approach to be efficient, different subsets of active nodes should be rotated successively. In a wireless sensor network, the concept of constructing a collection of disjoint dominating sets whose activity can be duty-cycled/rotated/scheduled is becoming increasingly as attractive as building a single minimum dominating set. In fact, having several disjoint dominating sets at the disposal of the sensor application can offer better fault-tolerance, load-balancing, and scalability as well as prolonged network lifetime. It also raises the possibility of catering to several quality of service requirements in terms of coverage accuracy, or traffic priorities.

1.1 Preliminaries

In this paper, we adopt the Unit Disk Graph (UDG) and Random Geometric Graph (RGG) models as defined in [20] to represent a wireless sensor network. Simply, random geometric graphs induce a probability distribution on unit disk graphs [6]. In our work, we use properties and approximation results stemming from both models. In general, the network is abstracted as an undirected graph $G = (V, E)$. We denote the number of nodes by $n = |V|$; the maximum degree is $\Delta$, the minimum degree is $\delta$, and average degree is $\overline{d}$. The distributed computation model we adopt is the standard synchronous message passing model, where time is split into discrete rounds. In each round, every node can perform some local computations, send a message to each neighbor, and receive messages from all neighbors. We assume that nodes exchange short messages of size $O(\log n)$ bits and a message sent in round $R$ arrives to its neighboring destination(s) before the next round $R + 1$ starts. All nodes start a computation synchronously and the time complexity of an algorithm is the number of rounds from the start until the last node terminates [30]. We also define a localized algorithm as an algorithm where each node operates solely on information that is available within a constant neighborhood of the node, typically the 1-hop neighborhood [17].

1.2 Related Work

Several recent works advocated the aforementioned strategy by putting it in the context of the domatic partition (DP) problem: an NP-hard graph theoretical problem whose objective is to find the largest number of disjoint dominating sets [20,15,25,27,28,19]. This number is upper bounded by $\delta + 1$. In [27], the authors define the maximum cluster-lifetime problem and propose a distributed approximation algorithm that finds a number of disjoint dominating sets within $O(\log n)$ of the optimal solution in arbitrary graphs. In [28], the authors propose the first centralized and distributed constant factor geometry-aware approximation algorithms to the domatic partition problem in Unit Disk Graphs. In [26], the authors propose a complex distributed topology-based solution to the connected domatic partition (CDP) problem in UDGs and describe a simple mechanism to rotate between the obtained disjoint connected dominating sets. Overall, the