Chapter 7

DETERMINATION OF CROSS SECTIONAL FORCES

7.1 Introduction

While we in chapter 6 focused exclusively on the determination of response displacements, we shall in this chapter deal with the determination of the corresponding cross sectional forces, i.e. the cross sectional stress resultants defined in chapter 1.3 (see Fig. 1.3.b). From a design point of view it is the maximum values of these quantities that decide the actual level of safety against structural failure. For a line like type of bridge structure the problem at hand is equivalent to that which is illustrated in Fig. 6.1, only that the response quantities we shall now set out to calculate are the cross sectional force components \( F \) (e.g. a bending moment, a torsion moment or a shear force) rather than the displacements which were in focus in chapter 6. The assumption of a Gaussian, stationary and homogeneous flow over the design period \( T \) (e.g. 10 min) is still valid, as well as the assumptions of linearity between load and load effects and a linear elastic structural behaviour. Thus, any cross sectional force component \( F \) may be described by the sum of its mean value and a fluctuating part that is Gaussian

\[
F_{tot}(x,t) = \bar{F}(x) + F(x,t)
\]  

(7.1)

The time domain chain of events is illustrated in Fig. 7.1.a. Similar to that which was argued for the determination of displacements, it is in the following taken for granted that the fluctuating part of the cross sectional response forces are quantified by their standard deviation \( \sigma_F \), as illustrated in Fig. 7.1.b. The maximum value of a force component at spanwise position \( x_r \) is then given by

\[
F_{\text{max}}(x_r) = \bar{F}(x_r) + k_p \cdot \sigma_F(x_r)
\]  

(7.2)

where \( k_p \) is the peak factor (that depends on the type of response process). The chain of events for cross sectional forces is equivalent to that which is shown for structural
displacements in Fig. 6.2 because the assumption of linear elastic structural behaviour implies that the relationship between structural displacements and cross sectional forces is also linear.

Thus, once the displacements have been determined, cross sectional forces may be obtained directly from the structural stiffness properties and the derivatives of the displacement functions according to usual structural mechanics procedures. While this is an appropriate strategy for the determination of the mean value $F$, it is not an advisable strategy for the determination of $\sigma_F$. There are two reasons for this. First, dynamic response displacements are in general obtained from a modal solution in frequency domain that contains a chosen number of eigen modes which have been obtained from an eigen value solution that is based on the distributed stiffness and mass properties of the structure. The standard deviation of the total response displacements are then built up of the sum of contributions from each of these modes, either in a mode by mode approach...