Chapter 4
Power Flow Analysis

This chapter describes the power flow analysis from both analytic and algorithmic viewpoints. Section 4.1 introduces the power flow problem through a simple example and clarifies the differences between power flow and circuit analysis. Section 4.2 provides a taxonomy of the power flow problem, while Section 4.3 presents the standard power flow equations. Section 4.4 describes the most common algorithms used for solving this problem. These are the Gauss-Seidel’s method, the Newton’s method and its variants, the fast decoupled power flow and the dc power flow. A discussion about the single and distributed slack bus models and a comparative example are also included in Section 4.4. Section 4.5 provides a general mathematical framework for the power flow problem based on the continuous Newton’s method. Finally, Section 4.6 summarizes the most relevant concepts provided in this chapter.

4.1 Background

A classical problem of circuit theory is to find all branch currents and all node voltages of an assigned circuit. Typical input data are generator voltages as well as the impedances of all branches. If all impedances are constant, the resulting set of equations that describe the circuit is linear.

For example, Figure 4.1 represents a single-phase steady-state ac system. Let us assume that the circuit shown in Figure 4.1 represents the single-phase equivalent of a symmetrical balanced three-phase transmission system where the branches between nodes 1, 2 and 3 are transmission lines, the impedance $\bar{z}_3$ is a load and the independent current sources $\bar{i}_1$ and $\bar{i}_2$ are generators. Assuming node 0 as the reference voltage, the current injections at nodes 1,

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1 In several papers and books, especially old ones, the power flow analysis is called load flow analysis. This notation should be avoided since, quoting Concordia and Tinney [33]: “Load does not flow, but power flows.”
2 and 3 are obtained based on the well-known branch current method as the solution of a simple set of linear equations:

\[
\begin{align*}
0 &= \frac{\bar{v}_1 - \bar{v}_2}{jx_{12}} + \frac{\bar{v}_1 - \bar{v}_3}{jx_{13}} - \bar{i}_1 \\
0 &= \frac{\bar{v}_2 - \bar{v}_1}{jx_{12}} + \frac{\bar{v}_2 - \bar{v}_3}{jx_{23}} - \bar{i}_2 \\
0 &= \frac{\bar{v}_3 - \bar{v}_1}{jx_{13}} + \frac{\bar{v}_3 - \bar{v}_2}{jx_{23}} - \bar{i}_3
\end{align*}
\]  

(4.1)

where \(\bar{i}_1\) and \(\bar{i}_2\) are imposed by the generators and \(\bar{i}_3\) depends on the voltage \(\bar{v}_3\), as follows:

\[
\bar{i}_3 = -\frac{\bar{v}_3}{\bar{z}_3}
\]

(4.2)

Rewriting (4.1) and (4.2) in vectorial form, one has:

\[
\begin{bmatrix}
\bar{i}_1 \\
\bar{i}_2 \\
0
\end{bmatrix} = 
\begin{bmatrix}
0 & 0 & 1/\bar{z}_3 \\
0 & 1/\bar{z}_3 & 0 \\
1/\bar{z}_3 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\bar{v}_1 \\
\bar{v}_2 \\
\bar{v}_3
\end{bmatrix} = \bar{Y}_{\text{tot}} \bar{v}
\]

(4.3)

where \(I_3\) is a 3 × 3 identity matrix and \(\bar{Y}\) is the so-called admittance matrix:

\[
\bar{Y} =
\begin{bmatrix}
1/jx_{12} + 1/jx_{13} & -1/jx_{12} & -1/jx_{13} \\
-1/jx_{12} & 1/jx_{12} + 1/jx_{23} & -1/jx_{23} \\
-1/jx_{13} & -1/jx_{23} & 1/jx_{13} + 1/jx_{23}
\end{bmatrix}
\]

(4.4)

\footnote{The mesh (or loop) current method is not used in this example because: (i) it can be used only for planar circuits, (ii) it is hard to implement in a computer code and, as a consequence of the previous issues, (iii) it has no relevant practical applications except for being a problem source for first-year students of electrical circuits.}