Chapter 3
LPV Systems and Representations

Abstract. In this chapter, a behavioral framework of LPV systems is introduced as an extension of the LTI behavioral approach. This is done with the intention to give a unified view on LPV system theory, that enables to approach LPV system identification in a well-founded system theoretic sense. First we define LPV dynamical systems from the behavioral point of view. Then we introduce the algebraic structure in which we formulate kernel, state-space, and input-output representations of LPV systems. We also analyze the properties of LPV systems in terms of state-observability, state-reachability, and dynamic stability.

3.1 General Class of LPV Systems

In this section, we establish the basics of a behavioral framework for linear parameter-varying (LPV) systems where a representation-free definition of such systems can be given and the previously considered concepts of LPV representations and corresponding theories can be re-established. Our main motivation is to set this framework as a tool for the analysis of LPV system identification in a well-founded sense.

One of the key concepts that is required to establish the LPV behavioral framework is an algebraic structure with elements describing differential equations, like $\mathbb{R}[\xi]$ (the ring of polynomials with real constant coefficients) used in the LTI case. As we will see, the required structure in the LPV case is based on polynomials with coefficients that are functions of the scheduling variable $p$ and its derivatives (continuous-time) or its time-shifts (discrete-time). The construction of this structure enables us to apply the results of the linear time-varying (LTV) behavioral approach, worked out by [239] and [78]. We use these results to establish three key theorems: the existence of kernel representations, the existence of state-kernel forms, and later in Sect. 3.2 the concept of left/right unimodular transformations. These theorems give the basic building blocks for the derivation of equivalence transformations between LPV representations, treated in Chap. 4 which have paramount significance for system identification.
First, let’s investigate what we call an LPV system from the behavioral point of view and what kind of physical phenomena are represented by this modeling concept.

### 3.1.1 Parameter Varying Dynamical Systems

In aerospace engineering, it is well-known that many airplanes, like the F-16 Fighting Falcon presented in Fig. 3.1, are nonlinear dynamical systems, but at a constant altitude they can be well approximated as an LTI system \([181, 42]\). Then, by viewing the aircraft as a collection of LTI behaviors corresponding to different altitude levels and using the altitude variable as a *scheduling* between them, we can arrive at an approximation of the global behavior. In this context, the concept of scheduling means the selection of the LTI behavior associated with a specific altitude level. This behavior describes the possible continuation of signal trajectories during the time interval in which the aircraft remains at the same altitude. Thus, the resulting representation of the global behavior involves coefficients that are functions of the scheduling. Such a modeling approach, that was introduced in Chap. 1 as the gain-scheduling principle, defines a parameter-varying (due to scheduling) and linear (in signal relation) system. Such systems are referred as LPV. However, it is important that an LPV system is more than just an array of LTI systems, because the governing scheduling rules or functions also define the dynamical behavior between each scheduling point, i.e. altitude points of this example. The concept of scheduling functions and “frozen” LTI behaviors provides an essential viewpoint on LPV systems which will be frequently used in the development of the identification approaches of Chap. 9.

In the general parameter-varying (PV) framework, the scheduling variable, commonly denoted by \(p\), is an external, so-called free signal of the system, that governs the dynamical behavior. From this aspect, the role of \(p\) can be understood as an other “time-variable” that determines the change of signal relations. However, the trajectory of \(p\) is unknown in advance which property distinguishes LPV systems from the LTV system class, where the variations of signal relations is directly associated with time. Based on this, the class of PV systems can be defined as follows:

**Definition 3.1 (Parameter-varying dynamical system).** A parameter-varying dynamical system \(S\) is defined as a quadruple

\[
S = (\mathbb{T}, \mathbb{P}, \mathbb{W}, \mathbb{B}), \tag{3.1}
\]

with \(\mathbb{T}\) the time-axis, \(\mathbb{P}\) the scheduling space with dimension \(n_p\), \(\mathbb{W}\) the signal space with dimension \(n_w\), and \(\mathbb{B} \subseteq (\mathbb{W} \times \mathbb{P})^\mathbb{T}\) the behavior (\(\mathbb{X}^\mathbb{T}\) is the standard notation for the collection of all maps from \(\mathbb{T}\) to \(\mathbb{X}\)).

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1 Note that systems where \(p\) is an internal variable (like output, input, or state) are called *quasi parameter-varying systems*. Still, such systems are commonly treated as a PV system with external scheduling variable, therefore in the upcoming analysis, \(p\) is assumed to be an independent variable. For more on quasi-PV systems, see Chap. 7.