Chapter 4

Uncertain Reliability Analysis

Uncertain reliability analysis was proposed by Liu [126] in 2010 as a tool to deal with system reliability via uncertainty theory. Note that uncertain reliability analysis and uncertain risk analysis have the same root in mathematics. They are separately treated for application convenience in practice rather than theoretical demand.

This chapter will introduce a definition of reliability index and provide some useful formulas for calculating reliability index.

4.1 Reliability Index

Reliability index is defined as the uncertain measure that some system is working.

Definition 4.1 (Liu [126]). Assume a system contains uncertain variables $\xi_1, \xi_2, \cdots, \xi_n$, and there is a function $R$ such that the system is working if and only if $R(\xi_1, \xi_2, \cdots, \xi_n) \geq 0$. Then the reliability index is

$$\text{Reliability} = \mathcal{M}\{R(\xi_1, \xi_2, \cdots, \xi_n) \geq 0\}. \quad (4.1)$$

Example 4.1: Consider a series system in which there are $n$ elements whose lifetimes are independent uncertain variables $\xi_1, \xi_2, \cdots, \xi_n$ with uncertainty distributions $\Phi_1, \Phi_2, \cdots, \Phi_n$, respectively. Such a system works if all elements are working, and the system lifetime $\xi$ has an uncertainty distribution $\Psi(x) = \Phi_1(x) \lor \Phi_2(x) \lor \cdots \lor \Phi_n(x)$. If we hope the system is working until time $T$, then the reliability index is

$$\text{Reliability} = \mathcal{M}\{\xi \geq T\} = 1 - \Phi_1(T) \lor \Phi_2(T) \lor \cdots \lor \Phi_n(T). \quad (4.2)$$

Example 4.2: Consider a parallel system in which there are $n$ elements whose lifetimes are independent uncertain variables $\xi_1, \xi_2, \cdots, \xi_n$ with uncertainty distributions $\Phi_1, \Phi_2, \cdots, \Phi_n$, respectively. Such a system works if there is at least one working element. Thus the system lifetime $\xi$ has an uncertainty distribution $\Psi(x) = \Phi_1(x) \land \Phi_2(x) \land \cdots \land \Phi_n(x)$. If we hope the system is working until time $T$, then the reliability index is
\[ \text{Reliability} = \mathcal{M}\{\xi \geq T\} = 1 - \Phi_1(T) \land \Phi_2(T) \land \cdots \land \Phi_n(T). \quad (4.3) \]

**Theorem 4.1** (Liu [126], Reliability Index Theorem). Assume \( \xi_1, \xi_2, \cdots, \xi_n \) are independent uncertain variables with uncertainty distributions \( \Phi_1, \Phi_2, \cdots, \Phi_n \), respectively, and \( R \) is a strictly increasing function. If some system is working if and only if \( R(\xi_1, \xi_2, \cdots, \xi_n) \geq 0 \), then the reliability index is

\[ \text{Reliability} = \alpha \quad (4.4) \]

where \( \alpha \) is the root of

\[ R(\Phi_1^{-1}(1 - \alpha), \Phi_2^{-1}(1 - \alpha), \cdots, \Phi_n^{-1}(1 - \alpha)) = 0. \quad (4.5) \]

**Proof:** It follows from Theorem 4.1.20 that \( R(\xi_1, \xi_2, \cdots, \xi_n) \) is an uncertain variable whose inverse uncertainty distribution is

\[ \Psi^{-1}(\alpha) = R(\Phi_1^{-1}(\alpha), \Phi_2^{-1}(\alpha), \cdots, \Phi_n^{-1}(\alpha)). \]

Since \( \text{Reliability} = \mathcal{M}\{R(\xi_1, \xi_2, \cdots, \xi_n) \geq 0\} = 1 - \Psi(0) \), we get (4.4).

**Theorem 4.2** (Liu [126], Reliability Index Theorem). Assume \( \xi_1, \xi_2, \cdots, \xi_n \) are independent uncertain variables with uncertainty distributions \( \Phi_1, \Phi_2, \cdots, \Phi_n \), respectively, and \( R \) is a strictly decreasing function. If some system is working if and only if \( R(\xi_1, \xi_2, \cdots, \xi_n) \geq 0 \), then the reliability index is

\[ \text{Reliability} = \alpha \quad (4.6) \]

where \( \alpha \) is the root of

\[ R(\Phi_1^{-1}(\alpha), \Phi_2^{-1}(\alpha), \cdots, \Phi_n^{-1}(\alpha)) = 0. \quad (4.7) \]

**Proof:** It follows from Theorem 4.1.25 that \( R(\xi_1, \xi_2, \cdots, \xi_n) \) is an uncertain variable whose inverse uncertainty distribution is

\[ \Psi^{-1}(\alpha) = R(\Phi_1^{-1}(1 - \alpha), \Phi_2^{-1}(1 - \alpha), \cdots, \Phi_n^{-1}(1 - \alpha)). \]

Since \( \text{Reliability} = \mathcal{M}\{R(\xi_1, \xi_2, \cdots, \xi_n) \geq 0\} = 1 - \Psi(0) \), we get (4.6).

**Theorem 4.3** (Liu [126], Reliability Index Theorem). Assume \( \xi_1, \xi_2, \cdots, \xi_n \) are independent uncertain variables with uncertainty distributions \( \Phi_1, \Phi_2, \cdots, \Phi_n \), respectively, and the function \( R(x_1, x_2, \cdots, x_n) \) is strictly increasing with respect to \( x_1, x_2, \cdots, x_m \) and strictly decreasing with respect to \( x_{m+1}, x_{m+2}, \cdots, x_n \). If some system is working if and only if \( R(\xi_1, \xi_2, \cdots, \xi_n) \geq 0 \), then the reliability index is

\[ \text{Reliability} = \alpha \quad (4.8) \]

where \( \alpha \) is the root of

\[ R(\Phi_1^{-1}(1 - \alpha), \cdots, \Phi_m^{-1}(1 - \alpha), \Phi_{m+1}^{-1}(\alpha), \cdots, \Phi_n^{-1}(\alpha)) = 0. \quad (4.9) \]