Abstract. The logic of comparative concept similarity $\mathcal{CSL}$ has been introduced in 2005 by Shemeret, Tishkovsky, Zakharyashev and Wolter in order to express a kind of qualitative similarity reasoning about concepts in ontologies. The semantics of the logic is defined in terms of distance spaces; however it can be equivalently reformulated in terms of preferential structures, similar to those ones of conditional logics. In this paper we consider $\mathcal{CSL}$ interpreted over symmetric and non-symmetric distance models satisfying the limit assumption, the so-called minspace distance models. We contribute to automated deduction for $\mathcal{CSL}$ in two ways. First we prove by the finite filtration method that the logic has the effective finite model property with respect to its preferential semantics. Then we present a decision procedure in the form of a labeled tableau calculus for both cases of $\mathcal{CSL}$ interpreted over symmetric and non-symmetric minspace distance models. The termination of the calculus is obtained by imposing suitable blocking conditions.

1 Introduction

The logics of comparative concept similarity $\mathcal{CSL}$ has been introduced in [7] to capture a form of reasoning about qualitative comparison between concept instances. In these logics we can express assertions or judgments of the form: “Renault Clio is more similar to Peugeot 207 than to Ferrari 430”. These logics may find an application in ontology languages, whose logical base is provided by Description Logics, allowing concept definitions based on proximity/similarity measures. For instance, the color “Reddish” may be defined as a color which is more similar to a prototypical “Red” than to any other color [7](in some color model as RGB). The aim is to dispose of a language where logical concept classification provided by standard DL is integrated with classification mechanisms based on calculation of proximity measures, typical for instance of domains like bio-informatics or linguistic. In this context, several languages comprising both absolute similarity measures and comparative similarity operator(s) have been considered [8]. In this paper we concentrate on the logic $\mathcal{CSL}$ which is obtained from the Boolean logic by adding one binary connective $\equiv$ expressing comparative similarity to a propositional language. In this language the above examples can be encoded (using a description logic notation) by:
Tableau Calculi for CSL over minspaces

(1) Reddish $\equiv \{\text{Red}\} \subseteq \{\text{Green}, \ldots, \text{Black}\}$

(2) Clio $\subseteq (\text{Peugeot207} \equiv \text{Ferrari430})$

Comparative similarity assertions such as (2) might not necessarily be the result of an objective numerical calculation of similarity measures, but they could be determined by the (integration of) subjective opinions of agents, answering, for instance, to questions like: “Is Clio more similar to Peugeot207 or to Ferrari 430?” In a more general setting, the language might contain several connectives $\equiv_{\text{Feature}}$ corresponding to a specific distance function $d_{\text{Feature}}$ measuring the similarity of objects with respect to each Feature (size, price, power, taste, color...).

The semantics of CSL is defined in terms of distance spaces, that is to say structures equipped by a distance function $d$, whose properties may vary according to the logic under consideration. In this setting, the evaluation of $A \equiv B$ can be informally stated as follows: $x \in (A \equiv B)$ iff $d(x, A) < d(x, B)$ meaning that the object $x$ is an instance of the formula $A \equiv B$ (i.e. it is more similar to $A$ than to $B$) if $x$ is strictly closer to $A$-objects than to $B$-objects according to the distance function $d$, where the distance of an object to a set of objects is defined as the *infimum* of the distances to each object in the set.

Properties of CSL with respect to different classes of models have been investigated in [7,13,9]. Moreover, CSL over arbitrary distance spaces can be seen as a fragment, indeed a powerful one (including for instance the logic $S4_u$ of topological spaces), of a general logic for spatial reasoning comprising different modal operators defined by (bounded) quantified distance expressions (namely the logic QML [9]). The satisfiability problem for the CSL logic (and in particular in the case of symmetric minspaces) is ExpTime-complete. Finally, when interpreted over subspaces of $\mathbb{N}^n$, $\mathbb{Z}^n$ or $\mathbb{R}^n$, it turns out that this logic is undecidable.

In this paper we consider the semantics of CSL induced by minspaces, that is to say distance spaces where the infimum of a set of distances is actually their minimum (the so-called limit assumption property). In this case, the logic CSL is naturally related to some conditional logics, whose semantics is often expressed in terms of preferential structures: that is to say possible-world structures equipped by a family of strict (pre)-orders $\prec_z$ indexed on objects/worlds [5].

Moreover, it has been shown that, under the limit assumption, CSL is able to distinguish between validity in symmetric and non-symmetric models.\(^1\) In this work we consider the logic CSL as defined over symmetric and respectively non-symmetric minspaces. The minspace property implies that the spaces have discrete distance functions. This requirement does not seem incompatible with the purpose of representing qualitative similarity comparisons, whereas it might not be reasonable for applications of CSL to spatial reasoning. The distinction between the symmetric and non-symmetric case is significant, for instance a kind of circular KB containing:

\(^1\) In contrast, in the general case where limit assumption is not assumed the logic cannot distinguish between symmetric and non-symmetric models.