Majority – Who Gets Elected Class Rep?

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Adam stares at the pile of papers in front of him. His class has just finished voting in the election of a class rep. All pupils have written the name of their preferred candidate on a piece of paper, and Adam has volunteered to count the votes and determine the result of the election. Prior to the election the class agreed that a candidate should become class rep only if more than half the class voted for him or her. If none of the candidates wins the absolute majority of the votes, the election will have to be repeated. Adam’s task is now to find out whether any candidate has received more than half of all the votes.

How should Adam approach this task? He doesn’t think about it much and decides to use the most straightforward method, namely putting down the names of all candidates that receive votes on a piece of paper, and keeping a tally of how many votes each of them has received. He picks up each of the ballot papers in turn to see which name has been written on it. If the name is not yet on his sheet, he writes down the name and puts one tally mark next to it. If the name is already on his sheet, he simply adds an extra tally mark next to that name. When Adam is done with all the ballot papers, his sheet looks as follows:

```
Shayna ++++
Liam   {{
James  {
Kate   }{
Kevin  ||
Hannah ++++ ++++-}||
Laura  {
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Now Adam looks for the name that has the most tally marks. If the number of those tally marks is larger than half the class size, that candidate is the winner of the election. Otherwise, none of the candidates has won the absolute majority, and the election needs to be repeated. Adam sees that
Hannah has received the most votes, namely 14. Adam’s class has 27 pupils. As 14 is more than 13.5 (half the class size), Hannah has won the absolute majority and is therefore elected class rep. Hannah has already been class rep in the previous year, and she will surely continue to do well in that role. All the pupils and the teacher congratulate Hannah on winning the election.

Later, Adam starts to think about the counting of the votes once more. He wonders how good the method that he had used was. For each ballot paper he had to look through the list of names on his sheet and add a tally mark or even a new name to the list. Doing this for 27 votes was fine, but in a larger election that would surely become quite tedious. Just imagine an election with hundreds or thousands of votes! The list of names could then also get very long, and consequently it would take quite long to check for the name on a ballot paper whether it is already in the list or not. Furthermore, Adam’s method of counting the votes has produced more information than was required: He didn’t just determine the winner of the election, but he also counted the votes for each of the other candidates. The latter information wasn’t really necessary for solving his task. Maybe it would have been possible to avoid determining unnecessary information and solve the task with less effort?

Let us remark here that it is also often important for data protection reasons to avoid unnecessary collection and processing of data and to generate only the information that is required to solve the task at hand; this significantly reduces the danger of misuse of personal data. A more detailed discussion of data protection issues is beyond the scope of this chapter, however.

Adam has recently become interested in algorithms and knows by now that there are often methods that are much faster than the most obvious method for solving a task. Therefore, he decides to investigate this further and find out whether there is a faster method for checking absolute majority in an election. Together with Laura, who is also interested in algorithms, Adam searches through various books on algorithms to see if they can find out something about the majority problem.

### Majority Algorithm

Laura and Adam indeed find information about the majority problem, i.e., the problem of determining from among $N$ given elements the majority element (the element that occurs more than $N/2$ times, if it exists). They come across the description of the following algorithm.