BIBO Stability of Spatial-temporal Fuzzy Control System

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Abstract. Three-dimensional fuzzy logic controller (3-D FLC) is a novel FLC developed recently for spatially-distributed systems. In this study, the BIBO stability issue of the 3-D fuzzy control system is discussed. A sufficient condition is derived and provided as a useful criterion for the controller design of the 3-D FLC. Finally, a catalytic packed-bed reactor is presented as an example of spatially-distributed process to demonstrate the effectiveness of the controller.

Keywords: Fuzzy control; Three-dimensional fuzzy set; Stability analysis; BIBO stability.

1 Introduction

Since the fuzzy control was first introduced and applied to a dynamic plant (steam engine and boiler combination) by Mamdani and Assilian [1], it has become one of the most active and fruitful research areas in fuzzy set theory. However, most of fuzzy controls [1],[2] in the real world are developed for lumped systems and only fewer are found for spatially-distributed systems. Actually, most of physical processes are spatially distributed systems, which are spatial-temporal systems with their states, controls, and outputs depending on the spatial position as well as the time. Such systems usually give rise to nonlinear control problems that involve the regulation of highly distributed control variables using spatially-distributed control actuators and sensors. Fuzzy control for spatially distributed systems becomes a challenging problem.

Recently, a novel three-dimensional fuzzy logic controller (3-D FLC), based on three-dimensional (3-D) fuzzy sets, has been developed for spatially-distributed systems [3]. The 3-D fuzzy set is composed of a traditional 2-D fuzzy set and a third dimension for spatial information; therefore, it has the inherent capability to express spatial information. The control strategy of the 3-D FLC is similar to how human operators or experts control the temperature in a space domain. The 3-D FLC has two obvious advantages over the traditional FLC. One is that the rules will not increase as sensors increase for spatial measurement, and the other is that the controller has inherent capability to process the spatial information. Thereby, the 3-D FLC has a great potential to a wide range of engineering applications for spatially-distributed processes.

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In this study, the stability issue of the 3-D fuzzy logic control system is discussed. Amount of efforts have been devoted to the stability analysis of conventional fuzzy control systems in the past decades [4]. The motivation of this study is the global stability analysis of conventional fuzzy two-term control system developed in [5] using the well-known small gain theorem which can result in a safe and more reliable stability region in practical application. Stimulated by it, the paper employs the small gain theorem for a global stability analysis for the 3-D fuzzy logic control system where the controlled process contains spatially distributed information. A sufficient condition is eventually derived and provided as a useful criterion for the parameter design of 3-D fuzzy two-term controller.

2 Preliminaries

In this section, the analytical model of 3-D fuzzy two-term controller is presented. The analytical model can be used for the stability analysis of the 3-D fuzzy two-term controller in the following sections.

Similar to the traditional two-term FLC, the 3-D two-term FLC is classified into two types: a PI type generates incremental control output $\Delta u$, and a PD type generates control output $u$. Contrary to its traditional counterpart, the input variables of the 3-D two-term FLC are functions of the spatial coordinates $Z$ and represents that the input information comes from the overall space domain. For instance, the two input variables are the spatial scaled error $e^s(z)$ and the spatial scaled error change $\Delta e^s(z)$. In actual applications, finite-point sensors can be used for measurement, therefore, $e^s(z)$ and $\Delta e^s(z)$ can be expressed in discrete form in space domain $Z = \{z_1, \cdots, z_q\}$, i.e., $e^s_i(z_i) = e^s_i$ and $\Delta e^s_i(z_i) = \Delta e^s_i$ ($i = 1, \cdots, q$), where $e^s_i \in E^s \subset IR$ and $\Delta e^s_i \in \Delta E^s \subset IR$ denote the scaled error and the scaled error change at the sensing location $z = z_i$ respectively, $E^s$ and $\Delta E^s$ denote the domains of $e^s_i$ and $\Delta e^s_i$ respectively, $IR$ denotes the set of all real numbers, and $q$ is the number of sensors.

![Fig. 1. Fuzzy sets for $e^s_i$ and $\Delta e^s_i$ (a) and for $\Delta u$ (b)](image)

Just like its traditional counterpart, the analytical model of the 3-D two-term FLC will be uniquely determined once its components are chosen. It is assumed that the