Design and Comparison of two Evolutionary Approaches for Solving the Rubik's Cube

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Abstract. Solutions calculated by Evolutionary Algorithms have come to surpass exact methods for solving various problems. The Rubik's Cube multiobjective optimization problem is one such area. In this paper we design, benchmark and compare two different evolutionary approaches to solve the Rubik's Cube. One is based on the work of Michael Herdy using predefined swapping and flipping algorithms, the other adapting the Thistlethwaite Algorithm. The latter is based on group theory, transforming the problem of solving the Cube into four subproblems. We give detailed information about realizing those Evolutionary Algorithms regarding selection method, fitness function and mutation operators. Finally, both methods are benchmarked and compared to enable an interesting view of solution space size and exploration/exploitation in regard to the Rubik's Cube.

1 Introduction

Since the Rubik's Cube's invention by Erno Rubik in 1974 and its commercialization in 1980 it has been the interest of not only hobbyists but also scientific research. Primarily mathematicians found themselves working on the Rubik's Cube as a discrete optimization problem trying to find efficient ways to solve it. With its simple structure the classic Rubik's Cube can reach a total number of $4.3 \cdot 10^{19}$ different configurations which induces an underlying complex optimization problem. To this day it is impossible to calculate all of those configurations. Also, the shortest length of sequences to reach any of those configurations (*God's Number*) is still unknown and subject to ongoing research [8]. In this paper we use two different evolutionary approaches to solve the Rubik's Cube as a discrete optimization problem. First, we briefly describe some characteristics and notation for the classic Rubik's Cube puzzle. We then introduce our two evolutionary approaches, one extending a method by Micheal Herdy [5], the other building upon Thistlethwaite's group-theoretic approach [2], [3], [10]. In contrast to [3] where we introduced our Thistlethwaite Evolution Strategy in detail, this work concentrates on our improvement of the Herdy Evolution Strategy and a thorough examination of both ES' mechanics. The careful benchmark and comparison of both algorithms provide an interesting analysis of solution space size and relation between exploration and exploitation in regard to the Rubik's Cube.

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2 The Rubik's Cube

2.1 Structure

The classic 3^3 Rubik's Cube is widely known and the one subject to this paper. It consists of 26 pieces: 8 corner pieces, 12 edge pieces and 6 center pieces, distributed equally on the six sides of the Cube. Each side of the Cube will be called *face*, each 2-dimensional square on a face will be referred to as *facelet*. Corners, edges and centers are all *cubies* - representing the physical object. A corner shows 3 facelets, an edge 2 and a center 1. Each side of the Rubik's Cube can be rotated clockwise (CW) and counterclockwise (CCW). Every such single move changes the position of 4 edges and 4 corners - note that the center facelet on every of the Cube's faces always stays in the same position. Thus, the color of a solved face is always determined by its center color. For each edge and corner it is of great importance to distinguish between *position* and *orientation*: i.e. an edge can be in its right position (defined by the two adjacent center colors) but in the wrong orientation (flipped).

2.2 Notation

There are several known notations [6] for applying single moves on the Rubik's Cube. We will use F, R, U, B, L, D to denote a clockwise quarter-turn of the front, right, up, back, left, down face and Fi, Ri, Ui, Bi, Li, Di for a counterclockwise quarter-turn. Every such turn is a *single move*. In Cube related research half-turns ($F2, R2, U2, B2, L2, D2$) are also counted as single move, we will do so as well. This notation is dependent on the users viewpoint to the cube rather than the center facelets' colors. However, as a convention used for this research work we assume the classic Rubik's Cube color configuration which is $white : yellow, red : orange, blue : green$ where : denotes $opposite\ of$. The starting orientation for the scrambles will be $F = white, R = red, U = blue, B = yellow, L = orange, D = green$.

2.3 Characteristics

Obviously the Rubik's Cube fulfills the characteristics of a mathematical group [4], [9]. The number of all attainable states $4.3 \cdot 10^{19}$ depicts the order of the Cube group $G_C =< F, R, U, B, L, D >$. All configuration of the Rubik's Cube can be reached by using combinations of single moves in this group, thus the single moves *generate* G_C. Further, there is always a neutral element, i.e. $F \cdot FFFF = FFFFF = F$ and $F^4 = 1$ (also showing the order of each generator in G_C is 4) and an inverse: $Fi \cdot F = 1$ and $Fi = FFF$.

The inverse of any operation is quickly calculated by reversing the order of single moves and their direction. For example the inverse of $FRiDLBUi$ would be $UBiLiDiRFi$. Further we can define subgroups of the group G_C. Let $H =< R, U >$ be a such a subgroup. If we only use moves from H there are just $2^6 \cdot 3^8 \cdot 5^2 \cdot 7 = 73483200$ different configurations we can attain [7]. This significantly