7 The Hotelling Paradox

Spatial Oligopoly

As mentioned in Chap. 1, the so-called Bertrand variant of oligopoly, with differentiated goods, is a less yielding format for interesting models than the very clear cut original Cournot case. The main reason is the difficulty of quantifying the difference between close substitutes. There is, however, one exception, and that is the 1929 model by Harold Hotelling. In this frame consumers regard the commodity supplied by different competitors as homogenous, but there is a difference between suppliers due to spatial distance from the consumers and the necessary transportation costs. Each producer has its market area, within which it is a monopolist. As the consumers always buy from the cheapest supplier, in terms of mill price plus transportation cost, there is competition at the market boundaries defined through the condition that supply prices break even. If one local monopolist raises mill price, it will see the market area diminished, and with this total demand for its supply.

This general idea is most fruitful, though its potential has been drowned in a paradox which also was suggested in the short Hotelling paper: If space is a finite one-dimensional line segment, and there are two competitors located in this segment, then, provided they can both choose location as well as mill price, the optima will be indeterminate. They will thus both locate in the middle of the segment, in stead of at the midpoints of each half-segment which would be socially optimal to the purpose of minimizing total transportation cost.

This paradox hides two different problems. One is the possibility of cutting out the competitor and taking the entire market, which remains a possibility for global optimum under any assumptions. But this is just the same problem as pointed out by Cournot’s critics; with both competitors in the same location nothing of the particularly spatial issues remains.
A different problem is that under Hotelling’s specific assumptions there do not exist any local inner optima, because the optimization problem is linear in the choice of location. In the lack of any inner optima with central location for each competitor in its proper market area, one is naturally thrown in the arms of the paradoxical cutting out situation as the only possibility.

So, what are Hotelling’s assumptions? As a matter of fact they are slightly contradictory. The boundaries of the market areas are established on the basis that no consumers buy from a supplier whose price including transportation cost is higher. Consumers are hence sensitive to price. But, on the other hand they always buy a fixed quantity independent of price. This leads to the linearity of the optimand with respect to location and the nonexistence of inner optima.

In his verbal discussion Hotelling indicated that making demand elastic to price would remove the problem, but he did not formalize this. Somewhat later, Lerner and Singer (1937) in an ingenious graphical analysis actually showed that just the assumption of a reservation price, above which no consumer would buy the commodity, removed the indeterminacy.

And Smithies (1941a, b) proposed a formal model using a linear demand function which he claimed would make the local optima determinate. But at the same time he argued that the integrations involved would be so messy that analysis would be almost impossible. Smithies’s formulation agrees with what Hotelling actually had in mind, but, as will be shown below, his fear for the technical difficulty of the analysis was much exaggerated.

It can, in fact, be shown formally that inner location and pricing optima exist, and one can even devise a dynamical stepwise search system for location and pricing that is convergent to location of the firms at the centres of each its monopoly area.

However, the cutting out possibility remains as a paradoxical possibility in the global sense, even when the inner solutions exist. One should perhaps also add that it would be interesting to know these facts in true two-dimensional geographical space. However, as always, spatial analysis in economics stops after the simplification to one dimension. In two dimensions, Smithies definitely is right, the integrations over market areas bounded by more or less complex boundaries become just too messy. Further, the present author has seen many attempts at generalizing the Hotelling problem to two dimensions, unfortunately none of them convincing. It is even difficult to imagine what the Hotelling problem generalized to two dimensions is. Perhaps three competitors in an equilateral triangle, or something similar. Anyhow, too complicated for formal analysis.