2.1 Introduction

Basic ideas concerning risk pooling and risk transfer, presented in Chap. 1, are progressed further in the present Chapter, mainly with the following purposes:

1. to discuss key features of premium calculation when non-homogeneous portfolios are concerned, namely portfolios consisting of risks with various claim probabilities;
2. to analyze, more deeply, the riskiness of a portfolio and the tools which can be used to face potential losses, in particular introducing the role of the shareholders’ capital;
3. to illustrate the possibility, for an insurance company, to transfer, in its turn, risk of losses to another insurer, namely the possibility to resort to reinsurance;
4. to address dynamic aspects of the management of insurance portfolios.

As we will see, the actions undertaken by an insurer in order to deal with potential losses (see points 1 and 3 above) constitute important examples of risk management actions, in the specific framework of insurance risk management.

The “basic” insurance cover, namely the cover related to Case 2 (Possible loss with fixed amount) widely used in Chap. 1, will still be addressed while dealing with the issues mentioned above, in order to keep the presentation at an acceptable level of complexity.
2.2 Rating: the basics

2.2.1 Some preliminary ideas

We refer to a portfolio of “basic” insurance covers, as defined in Chap. 1 (see, in particular Case 2 in Sects. 1.2.3, 1.4.2, 1.6.1, and 1.7.2), and we focus on the calculation of net premiums (i.e. not including loadings for expenses).

We assume that, for each risk, the premium is proportional to the benefit (that we also call the “sum insured”) paid in the case of a claim. Denoting (as in Chap. 1) with $x$ the benefit for the generic risk, the premium is then given by $x \bar{p}$, where the quantity $\bar{p}$ represents the premium for one monetary unit of benefit. In the insurance language, $\bar{p}$ is commonly called the *premium rate*.

The following are natural choices:

a. set $\bar{p}$ equal to the probability of a claim, $p$, as implied by the equivalence principle (see, for example, Sect. 1.7.4 and formula (1.7.3) in particular);

b. set $\bar{p}$ equal to the adjusted probability of a claim, $p'$, so that riskiness is accounted for via an implicit safety loading (see formula (1.7.14) in particular).

Although we now do not deal with implicit safety loadings, the first choice is not the only feasible one, as we will see in the next sections. Anyhow, the premium rate should reflect, at least to some extent, the probability of a claim. As a consequence, a number of premium rates, $\bar{p}_1, \bar{p}_2, \ldots$, should be used for calculating the premiums for risks with various claim probabilities. The set of rules which link the premium rates to the claim probabilities constitutes a *rating system*. The rating system is the basis underlying the construction of an *insurance tariff* (which also includes loading for expenses, possible discounts, and so on).

2.2.2 Homogeneous risks

First, we assume that the $n$ risks, which constitute the portfolio, are homogeneous in probability. As usual, we denote with $x^{(j)}$ the potential loss and hence the benefit for the $j$-th risk, and with $p$ the probability of loss for each of the insured risks.

According to the equivalence principle, the net premium for the $j$-th risk, $P^{(j)}$, is then given by

$$P^{(j)} = x^{(j)} p$$

(2.2.1)

Thus, the premium rate is given by the probability $p$.

At the portfolio level, the premiums expressed by (2.2.1) lead to the so-called *technical equilibrium* (clearly, in terms of expected value). Indeed, we have

$$\sum_{j=1}^{n} P^{(j)} = \sum_{j=1}^{n} x^{(j)} p = \mathbb{E}[X^{[p]}]$$

(2.2.2)