As a scientific concept, the so-called control stands for a special effect a controlling device exerts on controlled equipment. It is a purposeful and selective dynamic activity. A control system contains at least three parts, including a controlling device, controlled equipment, and an information path. A control system that is made up of only these three parts is known as an open loop control system, as shown in Figure 9.1. Each open loop control system is quite elementary, in which the input directly controls the output, with the fatal weakness that it does not have any resistance against disturbances.

A control system with a feedback return is known as a closed loop control system, as shown in Figure 9.2. The closed loop control system materializes its control through the combined effect of the input and the feedback of the output. One of the outstanding characteristics of closed loop systems is its strong ability to assist disturbances with their outputs constantly vibrating around the pre-determined objectives. So, closed loop control systems possess a certain kind of stability.
A grey control system stands for such a system whose control information is only partially known, and is ordinarily known as a grey system for short. The control of grey systems is different of that of the general white systems. It is mainly because of the existence of grey elements in the systems of concern. Under such conditions, one first needs to understand the possible connection between the systems’ behaviors and the parametric matrices of the grey elements, how the systems’ dynamics differ from one moment to the next, in particular, how to obtain a white control function to alter the characteristics of the systems and to materialize control of the process of change of the systems. Grey control contains not only the general situation of systems involving grey parameters, but also the construction of controls based on grey systems analysis, modeling, prediction, and decision-making. The thinking of grey control can deeply reveal the essence underneath the problems of interest and help materialize the purpose of control.

## 9.1 Controllability and Observability of Grey Systems

The concepts of controllability and observability are two fundamental structural characteristics of systems seen from the angle of control and observation. This section focuses on the problems of the controllability and observability of grey linear systems.

Assume that $U = [u_1, u_2, \cdots, u_j]^T$ is a control vector, $X = [x_1, x_2, \cdots, x_n]^T$ a state vector, and $Y = [y_1, y_2, \cdots, y_m]^T$ the output vector. Then

$$
\begin{align*}
\dot{X} &= A(\otimes)X + B(\otimes)U \\
Y &= C(\otimes)X
\end{align*}
$$

(9.1)

is known as the mathematical model of a grey linear control system, where $A(\otimes) \in G_{n,n}$, $B(\otimes) \in G_{n,j}$, $C(\otimes) \in G_{m,n}$. Correspondingly, $A(\otimes)$ is known as the grey state matrix, $B(\otimes)$ the grey control matrix, and $C(\otimes)$ the grey output matrix. In some studies, to emphasize that fact that $U$, $X$, and $Y$ change with time, the dynamic characteristic of the system, we also respectively write the control vector, state vector, and the output vector as $U(t)$, $X(t)$, and $Y(t)$.

The first group of equations

$$
\dot{X}(t) = A(\otimes)X(t) + B(\otimes)U(t)
$$

(9.2)

in the mathematical model of grey linear control systems in equ. (9.1) is known as the state equation, while the second group of equations

$$
Y(t) = C(\otimes)X(t)
$$

(9.3)

is known as the output equation.