Chapter 10  Blackout Model Based on AC Power Flow

Cascading blackout models based on optimal AC power flows are constructed in this chapter. Since these models are developed using SOC theory and incorporate the Manchester model, they can be utilized to describe the fast dynamics in cascading failures as well as the slow dynamics during the evolution of power grids. The advantage of this approach is that at both the macroscopic and microscopic levels, one can reveal and then study in depth the SOC characteristics in power grids.

From the practical engineering point of view, blackout models based on DC power flows have their own limitations because the computations for DC power flows cannot take into account the influence of reactive powers and voltages, while sometimes blackouts in real systems are caused exactly by voltage collapse following the shortage of reactive powers\(^{[1-4]}\). To deal with this limitation, the Manchester model\(^{[5]}\) has been proposed to simulate cascading failures and discuss several criticality criteria based on AC power flows. However, if we examine the Manchester model using SOC theory, it is clear that the model only simulates fast dynamic processes, but without the slow ones. Hence, the model is only related to criticality, but has few connections to the macroscopic self-organization. To overcome this, we develop cascading blackout models based on optimal AC power flows to disclose and then study in depth the SOC characteristics in power grids at both the macroscopic and microscopic levels in this chapter.

10.1 Mathematical Description of Optimal AC Power Flow

The key objective for the dispatch and management of power systems is to lower operation costs and promote social and economic benefits while maintaining the systems’ stability. To achieve this objective in a real power grid, the main tool is the computation of optimal power flows (OPF)\(^{[6]}\).

Mathematically speaking, the optimal power flow problem is a nonlinear programming problem, which can be formulated as follows. Consider a power system consisting of \(n\) buses and \(m\) lines. At time \(k\), the objective for the computation of the optimal power flows is to minimize the power loss in the system:

\[
\min F(P, Q) = \sum_{i \in G} P_{gi,k} - \sum_{j \in L} P_{dj,k} \tag{10.1}
\]
Power Grid Complexity

where $G$ is the set of generator buses, $L$ is the set of load buses, $P_{g_{i,k}}$ and $P_{d_{j,k}}$ are the active powers at generator bus $i$ and load bus $j$ respectively, and the vectors $P$ and $Q$ are the decision variables to be determined that correspond to the active and reactive power outputs of all the generators.

The optimization constraints include the power flow constraint, the generation output constraint, the bus voltage constraint and the line capacity constraint. Now we describe these constraints in detail.

1) Power flow constraint

$$P_{i,k} = V_{i,k} \sum_{j=1}^{n} V_{j,k} \left( G_{i,j,k} \cos \theta_{i,j,k} + B_{i,j,k} \sin \theta_{i,j,k} \right)$$ (10.2)

$$Q_{i,k} = V_{i,k} \sum_{j=1}^{n} V_{j,k} \left( G_{i,j,k} \sin \theta_{i,j,k} - B_{i,j,k} \cos \theta_{i,j,k} \right)$$ (10.3)

where $P_{i,k}$ and $Q_{i,k}$ are the injected active and reactive powers at bus $i$, $V_{i,k}$ and $\theta_{i,k}$ are the amplitude and phase of the voltage at bus $i$, $\theta_{i,j,k} = \theta_{i,k} - \theta_{j,k}$ is the difference in phase-angles of the voltages at bus $i$ and bus $j$, and $Y_k = (G_{i,j,k} + jB_{i,j,k})_{n \times n}$ is the admittance matrix of the system.

2) Generator output constraint

$$P_{g_{i,k}} \leq P_{g_{i,k}} \leq P_{g_{i,k}}^\text{max}, \quad i \in G$$ (10.4)

$$Q_{g_{i,k}} \leq Q_{g_{i,k}} \leq Q_{g_{i,k}}^\text{max}, \quad i \in G$$ (10.5)

where $P_{g_{i,k}}$ and $Q_{g_{i,k}}$ are the active and reactive powers outputs of generator $i$, $P_{g_{i,k}}^\text{max}$ and $Q_{g_{i,k}}^\text{max}$ are their corresponding upper limits respectively, and $P_{g_{i,k}}^\text{min}$ and $Q_{g_{i,k}}^\text{min}$ are their corresponding lower limits respectively.

3) Bus voltage constraint

$$V_{i,k}^\text{min} \leq V_{i,k} \leq V_{i,k}^\text{max}$$ (10.6)

where $V_{i,k}$ is the amplitude of the voltage at bus $i$, and $V_{i,k}^\text{max}$ and $V_{i,k}^\text{min}$ are its upper and lower limits respectively.

4) Line capacity constraint

$$-F_{l_{i,k}}^\text{max} \leq F_{l_{i,k}} \leq F_{l_{i,k}}^\text{max}, \quad l = 1, 2, \cdots, m$$ (10.7)

where $F_{l_{i,k}}$ is the power transferred through line $l$, and $F_{l_{i,k}}^\text{max}$ is the capacity of line $l$. 292