11 Introduction: Definitions and Concepts

Financial markets can be regarded from various points of view. Firstly there are economic theories which make assertions about security pricing; different economic theories exist in different markets (currency, interest rates, stocks, derivatives, etc.). Well known examples include the purchasing power parity for exchange rates, interest rate term structure models, the capital asset pricing model (CAPM) and the Black-Scholes option pricing model. Most of these models are based on theoretical concepts which, for example, involve the formation of expectations, utility functions and risk preferences. Normally it is assumed that individuals in the economy act ‘rationally’, have rational expectations and are averse to risk. Under these circumstances prices and returns can be determined in equilibrium models (as, for example, the CAPM) which clear the markets, i.e., supply equals aggregate demand. A different Ansatz pursues the arbitrage theory (for example, the Black-Scholes model), which assumes that a riskless profit would be noticed immediately by market participants and be eliminated through adjustments in the price. Arbitrage theory and equilibrium theory are closely connected. The arbitrage theory can often get away with fewer assumptions, whereas the equilibrium theory reaches more explicitly defined solutions for complex situations.

The classic econometric models are formulated with economically interpreted parameters. One is interested in the following empirical questions:

1. How well can a specific model describe a given set of data (cross section or time series)?
2. Does the model help the market participants in meeting the relative size of assertions made on future developments?
3. What do the empirical findings imply for the econometric model? Will it eventually have to be modified? Can suggestions actually be made which will influence the function and structural organisation of the markets?

In order to handle these empirical questions, a statistical inquiry is needed. Since, as a rule with financial market data, the dynamic characteristics are
the most important, we will mainly concentrate on the time series analysis. First of all, we will introduce the concepts of univariate analysis and then move to the multivariate time series. The interdependence of financial items can be modelled explicitly as a system.

Certain terms, which are often used in time series analysis and in the analysis of financial time series, are introduced in a compact form. We will briefly define them in the next section.

11.1 Some Definitions

First we will need to look closer at stochastic processes, the basic object in time series analysis.

**Definition 11.1 (stochastic process)**
A stochastic process $X_t$, $t \in \mathbb{Z}$, is a family of random variables, defined in a probability space $(\Omega, \mathcal{F}, P)$.

At a specific time point $t$, $X_t$ is a random variable with a specific density function. Given a specific $\omega \in \Omega$, $X(\omega) = \{X_t(\omega), t \in \mathbb{Z}\}$ is a realisation or a path of the process.

**Definition 11.2 (cdf of a stochastic process)**
The joint cumulative distribution function (cdf) of a stochastic process $X_t$ is defined as

$$F_{t_1, \ldots, t_n}(x_1, \ldots, x_n) = P(X_{t_1} \leq x_1, \ldots, X_{t_n} \leq x_n).$$

The stochastic process $X_t$ is clearly identified, when the system of its density functions is known. If for any $t_1, \ldots, t_n \in \mathbb{Z}$ the joint distribution function $F_{t_1, \ldots, t_n}(x_1, \ldots, x_n)$ is known, the underlying stochastic process is uniquely determined.

**Definition 11.3 (conditional cdf)**
The conditional cdf of a stochastic process $X_t$ for any $t_1, \ldots, t_n \in \mathbb{Z}$ with $t_1 < t_2 < \ldots < t_n$ is defined as

$$F_{t_n|t_{n-1}, \ldots, t_1}(x_n \mid x_{n-1}, \ldots, x_1) = P(X_{t_n} \leq x_n \mid X_{t_{n-1}} = x_{n-1}, \ldots, X_{t_1} = x_1).$$

Next we will define moment functions of the real stochastic process. Here we will assume that the moments exist. If this is not the case, then the corresponding function is not defined.