Chapter 4
Processing Time Variability

In the previous chapter, an approximation for the cycle time in a system queue was developed (or waiting time in the queue for a machine). The relationship consists of four parameters. These are the squared coefficient of variation of the inter-arrival time process \( C^2_a \), the squared coefficient of variation of the service time process \( C^2_s \), the machine utilization \( u \), and the mean service time \( E[T_s] \). This relationship is

\[
CT_q(G/G/1) = \frac{(C^2_a + C^2_s)}{2} \left( \frac{u}{1-u} \right) E[T_s] .
\]  

(4.1)

From this relationship, it is clear that reducing one of the variability components, \( C^2_a \) or \( C^2_s \), will reduce the cycle time in the queue. What might be overlooked is that reducing variability is equivalent to reducing the machine utilization by some factor with respect to the mean cycle time measure. In more direct terms, reducing process variability is equivalent to finding extra capacity in the system since a reduction of utilization with a constant arrival rate implies an increase in the mean processing rate.

To illustrate the equivalence between reducing variability and utilization, consider a single machine system with the following parameter values:

\[
\begin{align*}
C^2_a &= 1 \\
C^2_s &= 1 \\
u &= 0.8 \\
E[T_s] &= 2 \text{ hr} .
\end{align*}
\]

Thus the cycle time in the queue \( CT_q \) is thus

\[
CT_q = \frac{(1 + 1)}{2} \left( \frac{0.8}{1-0.8} \right) 2 \text{ hr} = 8 \text{ hr} .
\]

Now if \( C^2_s \) is reduced by 10% to 0.9, the resulting cycle time is 7.6 hours, a reduction of 5%. It would take a reduction in machine utilization from 80% to 79.17%
to accomplish this same cycle time decrease if $C_i^2$ was not changed. Thus, reducing service time variability (or inter-arrival time variability) has the same effect as obtaining additional machine capacity. The equivalent utilization factor $u$ is found by solving the equation

$$\frac{(1+1)}{2} \left( \frac{u}{1-u} \right)^2 = 7.6,$$

$$4.6u = 3.6,$$

$$u = 0.7917.$$

Now a 50% reduction in the service time variability for this example data would reduce the cycle time measure to 6 hours. The equivalent machine utilization factor for 6 hours given the original system parameters is 0.75. This is a reduction in utilization, or the mean service time, of 6.25%. Either of these changes would result in a cycle time in the queue of 6 hours which is a 25% reduction from the original 8 hours.

The conclusion that can be drawn from this analysis is that reducing component variability is equivalent to increasing system capacity when measured by cycle time response. So it is very important to concentrate on reducing variability for the inter-arrival and service time processes since these reductions are like finding “free” machine capacity.

There are many factors that contribute to the variability of the length of time that a job spends in processing. The term “in processing” indicates that the job has control of the machine and other jobs cannot be processed until this job is completed. Job residence time includes the actual time that the machine is processing the job (herein called the natural processing time to distinguish it from the total time on the machine), any setup needed to place the job on the machine and prepare the machine for the particular job type, any delay due to the unavailability of an operator once the machine is available for allocation to that specific job, and delays due to machine breakdowns and repairs. Scheduled maintenance is normally accounted for in the available machine time rather than accounting for this lost time as part of a specific job’s residence time. The principle contributors to job residence time variability are:

- Natural processing time variability — the variability evident in the time it takes to actually process a specific job type.
- Random breakdowns and repairs during processing — the variability of the time between breakdowns and the variability of the time to repair a broken machine.
- Operator unavailability can induce random delays in the time a job spends “in control of” a machine. This time delay occurs when a machine and job are available with the operator being needed to setup the machine and start processing, but the operator is busy serving another machine/job combination.
- Job class setup and take-down times — the time caused by a job-type change on a machine. This change-over time generally occurs at the end of processing of one job type and the starting of a different job class.