Chapter 14
Regulation of Traffic

14.1 Introduction

The advent of techniques to measure velocities of probe vehicles using GPS technology, for instance, complementing or replacing fixed sensing infrastructures such as density sensors of the road traffic sensors, motivates the revision of conceptual, mathematical algorithms and software based models used by the transportation engineering community. Bruce Greenshields used in 1933 photographic measurement methods for the first time to describe a phenomenological law described by a quadratic relation between vehicles and their density and flows, called the fundamental diagram.

![Fundamental Diagrams](image)

Fig. 14.1 Original 1933 Greenshields’ Fundamental Diagrams. The two first diagrams are the historical Greenshields’ diagrams.

Later, in the middle of the years 1950, Lighthill, Whitham and Richards, proposed a partial differential equation (conservation law) with concave flow function, the solution of which is the density of traffic at each time and at each position (see Chap. 16, p. 631).
This attempt to characterize the behavior of congested traffic became the seminal model for numerous highway traffic flow studies available in the traffic engineering literature today.

Thirty five years later, Gordon Newell introduced the concept of “cumulative number” of vehicles passing at given position after a given time on a one-dimensional road, since density was the prevalent concept at the time. He acknowledged that Karl Moskowitz, an engineer from the California Department of Transportation who did not bother to publish, used this concept for some time to investigate properties of traffic.

It is thus convenient to call this specific Hamilton–Jacobi equation the “Moskowitz equation”. Since data from the sensors can be written as Cauchy or Dirichlet boundary conditions, they were sufficient to determine a solution by classical methods.

Next, J.C. Luke and Newell discovered that these cumulative number functions are solutions to this Moskowitz equation and a variational principle. Daganzo, who took over these equations for studying them mathematically in 2004, wrote: “Luke (1973) and Newell (1993) proposed the minimum operation as a way of selecting the unique and correct value at every point in space-time without proving it. It should be remembered in this respect that a “correct”, i.e., physically meaningful, solution of the problem [...]”.

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