Chapter 7
Management of Renewable Resources

7.1 Introduction

This chapter is devoted to some problems dealing with births, growth, and survival of populations.

Section 7.2, p. 248 starts with a simple model of the growth of the biomass of a renewable resource, fish population, for instance, following the track of Sect. 6.4, p. 207 of Chap. 6. We first review some dynamical growth systems devised from Malthus to Verhulst and beyond, up to recent growth models. They all share the same direct approach (see Box 1, p. 5) where growth models are proposed and the evolutions they govern are studied.

Instead, we suggest to follow the inverse approach to the same problem in the framework of this simple example (see Box 2, p. 5): we assume only the knowledge or viability constraints and inertia thresholds. This provides, through the inertia function, the “Mother of all viable feedbacks” (since the term “matrix” is already used) governing the evolutions satisfying these two requirements. Among them, we find the Verhulst feedback driving the logistic curves, the trajectories of which are “S-shaped”. Furthermore, we are tempted to look for the two “extreme feedbacks”. Each of them provides what we call inert evolutions, with velocity of the growth rates equal to the inertial threshold. When, alternatively combined (amalgamated), they govern cyclic viable evolutions. This may provide explanations of economic cycles and biological cycles and clocks without using periodic systems.

Combined with the Malthusian feedback, which does not belong the progeny of this Mother, we obtain heavy evolutions, the growth rate of which is kept constant (Malthusian evolution) as long as possible and then, switched to an inert evolution.

Section 7.3, p. 262 introduces fishermen. Their industrial activity depletes the growth rate of their renewable resource. They thus face a double challenge: maintain the economic viability of their enterprise by fishing enough resources, and not fishing too much for keeping the resource alive, even
though below the level of economic viability. Without excluding the double catastrophe: fishes disappear forever.

Even though the real problem is complex, involving too many variables to be both computable and reasonably relevant, the illustration of the concepts of permanence kernels and crisis functions in the framework of this simple example is quite instructive. It offers a mathematical metaphor showing that these two types of constraints, economical and ecological, produce evolutionary scenarios which can be translated in terms of viability concepts, sharing their properties.

### 7.2 Evolution of the Biomass of a Renewable Resource

We illustrate some of the results of Sect. 6.4, p. 207 with the study, for example, of the evolution of the biomass of one population (of renewable resources, such as fish in fisheries) in the framework of simple one-dimensional regulated systems. The mention of biomass is just used to provide some intuition to the mathematical concepts and results, but not the other way around, as a “model” of what happens in this mysterious and difficult field of management of renewable resources. Many other interpretations of the variables and controls presented could have been chosen, naturally. In any case, whatever the chosen interpretation, these one-dimensional systems are far too simplistic for their conclusions to be taken seriously.

We assume that there is a constant supply of resources, no predators and limited space: at each instant \( t \geq 0 \), the biomass \( x(t) \) of the population must remain confined in an interval \( K := [a, b] \) describing the environment (where \( 0 < a < b \)). The maximal size \( b \) that the biomass can achieve is called the carrying capacity in the specialized literature.

The dynamics governing the evolution of the biomass is unknown, really.

#### 7.2.1 From Malthus to Verhulst and Beyond

Several models have been proposed to describe the evolution of these systems. They are all particular cases of a general dynamical system of the form

\[
x'(t) = \tilde{u}(x(t))x(t)
\]  

(7.1)

where \( \tilde{u} : [a, b] \to \mathbb{R} \) is a feedback, i.e., a mathematical translation of the growth rate of the biomass of the population feeding back on the biomass (specialists in these fields prefer to study growth rates than velocities, as in mechanics or physics).