Phenomenology of Neutrino Oscillations

So far a number of different neutrino experiments have convincingly observed the phenomena of neutrino oscillations, indicating that neutrinos have rest masses and lepton flavors are mixed. This is the first compelling experimental evidence for new physics beyond the standard model (SM) of elementary particle physics. In this chapter we shall first describe the basic properties of neutrino oscillations in vacuum and then explain the Mikheyev-Smirnov-Wolfenstein (MSW) mechanism for neutrino flavor conversions in matter. Section 5.2 is devoted to an analysis of quantum coherence in neutrino oscillations. We shall reformulate neutrino oscillations by means of the language of the density matrix and flavor polarization vector in Section 5.3. Some ongoing and future accelerator- and reactor-based neutrino oscillation experiments will be briefly introduced in Section 5.4.

5.1 Neutrino Oscillations and Matter Effects

Soon after the discovery of electron antineutrinos in 1956, Bruno Pontecorvo postulated that there might exist the phenomenon of neutrino-antineutrino oscillations by analogy with the phenomenon of $K^0$-$\bar{K}^0$ mixing (Pontecorvo, 1958). Shortly after the discovery of muon neutrinos in 1962, Ziro Maki, Masami Nakagawa and Shoichi Sakata proposed that two different neutrino flavors could mix with each other and thus the phenomenon of $\nu_e \leftrightarrow \nu_\mu$ or $\bar{\nu}_e \leftrightarrow \bar{\nu}_\mu$ transitions might take place (Maki et al., 1962). Their ideas point to the concept of neutrino (flavor) oscillations: given a beam of neutrinos in a definite flavor state at the source, one may find them in another flavor state at the detector which is at a certain distance from the source. This kind of quantum phenomenon can naturally occur if three known neutrinos have non-degenerate masses and lepton flavors are mixed. Neutrino oscillations have been well established in the past twelve years, thanks to a number of solar, atmospheric, reactor and accelerator neutrino experiments. Both the
longstanding solar neutrino puzzle and the observed deficit of atmospheric muon neutrinos are actually attributed to neutrino oscillations. In this section we first outline the basic properties of neutrino oscillations in vacuum and then describe the phenomenology of neutrino oscillations in matter.

5.1.1 Neutrino Oscillations in Vacuum

Three neutrinos ($\nu_e, \nu_\mu, \nu_\tau$) are defined as the flavor eigenstates, which accord with three charged leptons ($e, \mu, \tau$), in their production processes via the weak charged-current interactions. Since neutrinos are assumed to be massless in the SM, their flavor and mass eigenstates coincide with each other and thus lepton flavors are conserved. But current neutrino oscillation experiments have demonstrated that neutrinos are massive and the SM is incomplete. Beyond the SM we define the mass eigenstates of three neutrinos as ($\nu_1, \nu_2, \nu_3$), whose eigenvalues are denoted by ($m_1, m_2, m_3$). In the basis where the mass eigenstates of charged leptons are identified with their flavor eigenstates, the phenomenon of neutrino mixing can be described by a $3 \times 3$ unitary matrix $V$, the so-called Maki-Nakagawa-Sakata (MNS) matrix (Maki et al., 1962):

$$
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau \\
\end{pmatrix} =
\begin{pmatrix}
V_{e1} & V_{e2} & V_{e3} \\
V_{\mu1} & V_{\mu2} & V_{\mu3} \\
V_{\tau1} & V_{\tau2} & V_{\tau3} \\
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3 \\
\end{pmatrix}.
$$

Different parametrizations of $V$ have been presented in Section 3.5.1.

Now we explain how neutrinos change their flavors when propagating in vacuum. A freely propagating neutrino should be on the mass-shell, or equivalently in the mass eigenstate $\nu_k(t, x)$ (for $k = 1, 2, 3$). Therefore, they must obey the Dirac equation ($i\partial - m_k$) $\nu_k(t, x) = 0$ and the Klein-Gordon equation $(\partial^2 + m_k^2) \nu_k(t, x) = 0$ (in the neglect of the spinor structure of the neutrino field which is essentially irrelevant to neutrino oscillations). For a stationary neutrino source, which is the case for all the existing neutrino oscillation experiments, the energy spectrum of neutrinos is fixed and one actually measures their propagation in space rather than in time. To be specific, let us assume that a neutrino beam with energy $E$ is propagating in the one-dimensional space $x$. In this case the corresponding Klein-Gordon equation can be rewritten as $[-(E + i\partial_x)(E - i\partial_x) + m_k^2] \nu_k(t, x) = 0$. In view of $-i\partial_x \nu_k(t, x) = p_k \nu_k(t, x)$ due to $\nu_k(t, x) \propto e^{-i(Et - p_k x)}$ for free neutrinos, we further obtain

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1 For neutrinos from the conventional sources, their oscillations have been treated in three different ways: the evolution of neutrino states in time, in space, or in space and time. They all lead to the same oscillation probabilities in the relativistic approximation $t \approx x$, as given in Eq. (5.5). However, the scheme of evolution in space is exclusively appropriate to understand the recent proposal of Mössbauer neutrino oscillations (Akhmedov et al., 2008, 2009).