Signal/system and detection/estimation theory is the most important theoretical and applied basis of modern information theory.

Distinguishing signal from background noise and estimating the parameters of detected signal are among the most important objectives of radar, sonar and communication systems\textsuperscript{[11-25]}. In particular, interference suppression in the detection of weak signal is the main task of many applications.

In sonar signal processing, the first objective of active or passive sonar is to identify the existence of the target. For active sonar, after the transmission of an active sonar signal, the receiver waits to receive the echo signal from the target. But the echo signal is usually distorted and suffers interference from various noises. For passive sonar, because the radiated noise from the target is unpredictable, the detection of signal is more uncertain\textsuperscript{[11-20,26-32]}.

The estimation of signal parameters is also an important task in sonar signal processing. The parameters include information about signal amplitude, phase, frequency and time delay. The randomness of signal and background noise make parameter estimation more difficult. It is possible to estimate the signal parameters with a certain level of accuracy. Therefore, probability theory and mathematical statistics have become the theoretical basis of signal detection and estimation theory\textsuperscript{[33-38]}.

In this chapter, the traditional theories of signal/system and detection/estimation are described with some examples of applications. The basic framework is “detection before estimation” rather than “joint detection and estimation” or so-called “estimation before detection”\textsuperscript{[39-42]}.

### 3.1 Some Basic Results from Probability Theory and Mathematical Statistics

Probability theory and mathematical statistics are basic branches of mathematics: the contents of these two branches are extensive and profound. It is impossible to
introduce the framework of this theory in this section, or even the most important results. The only aspects that we are going to present are the basic conclusions from some study areas that are already used in signal detection and estimation. The reader can then understand how to use these results in sonar signal processing and sonar design. Readers who are interested in the detailed theoretical background of probability theory and mathematical statistics can read a relevant monograph in this area. Probability theory and mathematical statistics are essential courses for undergraduate or graduate students who wish to study signal processing and its applications.

### 3.1.1 Basic Definition of Probability

Probability theory is a branch of mathematics concerned with the analysis of random phenomena. The central objects of probability theory are random variables, stochastic processes, and events: mathematical abstractions of non-deterministic events or measured quantities that may either be single occurrences or evolve over time in an apparently random fashion. As a mathematical foundation for statistics, probability theory is essential to many human activities that involve quantitative analysis of large sets of data. Methods of probability theory are also applied in the description of complex systems given only partial knowledge of their state, as in statistical mechanics. A great discovery of 20th century physics was the probabilistic nature of physical phenomena at atomic scales, as described in quantum mechanics\(^{[43]}\). Mathematical statistics is the study of statistics from a purely mathematical standpoint, using probability theory as well as other branches of mathematics such as linear algebra and analysis. It deals with gaining information from data. In practice, data often contain some randomness or uncertainty. Statistics handles such data using methods of probability theory.

In the natural world, under certain conditions the outcomes of many events can happen or not happen. This kind of event is called a random event. For example, a coin with two faces: value and image. In each throw, the value face and image face have an equal chance of facing upwards but it is impossible to predict which face will face upwards in a single throw. Therefore the event “value face upward in one throw” is a random event.

Denote the all possible outcomes of an experiment by \(X\): this is called the sample space. Each point in \(X\), i.e., one possible outcome, is called a basic event. A random event is a set consisting of some of the points. Therefore, set theory in mathematics can be used to describe and discuss random events.

**Probability of an Event**

Probability is a quantity used to describe the value of the possibility of occurrence of an event. The precise definition of probability depends on the application area.