Chapter 17
Constitutive Models of Mechanical Behavior of Media with Stress State Dependent Material Properties

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Abstract The behavior of heterogeneous materials is studied. The dependence of the effective elastic properties of micro-heterogeneous materials on the loading conditions are analyzed and corresponding mathematical methods for the description of the observed effects are proposed. The constitutive relations of the theory of elasticity for isotropic solids with stress state dependent deformation properties are considered. The possible approach to the formulation of the constitutive relations for the elastic anisotropic solids that elastic properties depend on the stress state type is considered, and the corresponding constitutive relations are proposed. The method for the determination of material’s functions on the base of experimental data is proposed. The quite satisfactory correspondence between the theoretical results and experimental data is shown.


17.1 Introduction

The experimental studies of deformation properties of many heterogeneous and composite materials display the dependence of their properties on the conditions of loading. There are different mechanisms related to this phenomenon. In the case of granular porous materials, the area of contact between the particles increases under compressive loads. Then one would expect that the elastic characteristics would increase under compression in comparison with values corresponding to the action...
of tensile loads. In the case of cracked materials, the crack opening occurs under tensile load and the effective cross section carrying the load is less than in a solid material. Therefore the effective deformation properties depend on the concentration of microcracks. Under the conditions of compressive loads, it is possible that the closure of microcracks and the contact of crack faces would happen. The mechanical properties in this case depend on the conditions of interactions on the crack faces that are determined in one’s turn by the ratios between different components of the stress tensor. This applies equally to an arbitrary type of loading. It means that the material properties are not invariant to the type of external forces but depend on the stress state type. For example, the initial slope of the stress-strain curve under conditions of compression is from 1.3 to 2 times the initial slope of the curve for tension [8].

Similar results have been obtained for structural graphite [1]. The effective stress-strain curves of ARV graphite are shown in the Fig. 17.1, which were obtained by a proportional loading of tubular specimens under plane stress conditions. The effective stress is $\sigma_0 = \sqrt{\frac{3}{2} S_{ij} S_{ij}}$, where $S_{ij} = \sigma_{ij} - \sigma \delta_{ij}$ is the stress deviator and $\sigma = \frac{1}{3} \sigma_{ii}$ is the hydrostatic component of the stress. The effective strain is $\varepsilon_0 = \sqrt{\frac{2}{3} e_{ij} e_{ij}}$, where $e_{ij} = \varepsilon_{ij} - \frac{1}{3} \varepsilon \delta_{ij}$ is the strain deviator and $\varepsilon = \varepsilon_{ii}$ is the bulk strain. Curves 1, 2, 3 and 4 correspond to uniaxial tension, uniaxial compression, shear and uniform biaxial tension, respectively. Instead of the single curve, as usually supposed in different theories of deformation, there is a fan of effective stress-strain curves and their deviation is noticeable. The curves have a weak non-linearity and a linear approximation of them is possible in a certain deformation range. Similar effects can be demonstrated for rocks, concrete, cast-iron and other materials [4].

The opposite effect sometimes can be observed in the case of composites. The fabric based carbon-carbon composites or composites with triaxial weave usually have considerable porosity. The fibers are tightened up under tension but they can buckle into the pores space under compression. The mechanisms of deformation are quite different for these loading conditions. The bending stiffness of fibers is much