A Projection Access Scheme Based on Prime Number Increment for Cone-Beam Iterative Reconstruction

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Abstract. The convergence speed of algebraic reconstruction technique (ART) depends heavily on the order in which the projections are considered. In this study, a projection access scheme based on prime number increment is proposed, which is applicable to uniform projection sampling in any angle range. We compared the results reconstructed from the proposed method with the results reconstructed from the conventional sequential method, the prime number decomposition method and random ordering method, for cone-beam X-ray computed tomography reconstruction and for the case of circular acquisition. The results indicate that using the proposed method can accelerate the convergence of ART and produces more accurate images with fewer artifacts.

Keywords: CT reconstruction, ART, projection order, prime number increment.

1 Introduction

Cone-beam x-ray computed tomography (CT) is one of the most important non-invasive imaging techniques. In the x-ray CT reconstruction, a volumetric image of object is reconstructed from the projection data. There are two major categories of CT image reconstruction: analytic and iterative methods. Iterative methods such as algebraic reconstruction technique (ART) [1] and expectation maximization (EM)[2-3] are superior to analytic methods such as the FDK [4] and the Katsevich algorithms [5] in handling incomplete and noisy projection data.

A relatively high demand for computational time is the main drawback to use iterative methods. Several approaches have been developed to accelerate the computation of iterative methods. One method is using the optimized ordering schemes for ART. Several methods have been proposed and evaluated so far [6–9].

The aim of this study is to design a “good” permutation of the ordering of the projection view’s index using the simple method. This paper is organized as follows. A brief review of ART and the proposed projection access system based on prime
number increment are given in Section 2. Section 3 describes the experiments, quantitative evaluation metrics, and the results. The conclusion is given in section 4.

2 Methods

2.1 Algebraic Reconstruction Techniques

The image reconstruction problem in CT can be modeled by the following equation:

\[ AX = Y \]  \hspace{1cm} (1)

where \( Y = (y_1, y_2, \cdots, y_I)^T \) is the projection data and \( I \) is the total number of projection rays, \( X = (x_1, x_2, \cdots, x_J)^T \) is an unknown image and \( J \) is the total number of the voxel in the image, and \( A = (a_{ij})_{I \times J} \) is the projection matrix and \( a_{ij} \) is the length of projection ray \( i \) through voxel \( j \). The problem is to reconstruct the \( X \) from the \( Y \).

The ART algorithm provides an efficient iterative way to solve the problem. It can be written as:

\[ x_j^{(k+1)} = x_j^{(k)} + \lambda_k \cdot \frac{y_i - \sum_{j=1}^{J} a_{ij} x_j^{(k)}}{\sum_{j=1}^{J} a_{ij}^2} \cdot a_{ij}. \]  \hspace{1cm} (2)

where \( i = k \mod(I) + 1 \) and \( \lambda_k \) is the relaxation parameter. This method was originally discovered by Kaczmarz in [10].

2.2 A projection Access System Based on Prime Number Increment

Denoting \( M \) as the total number of projections and the projections \( P_i, 0 \leq i \leq M - 1 \), are assumed to be equally spaced by an angle \( \varphi = \frac{2\pi}{M} \) in the interval \([0,2\pi)\). In the following, we will give the permutation \( \tau \) of the ordering of the \( M \) projections \( \{1, 2, \cdots, M\} \) using prime number increment:

\[ \tau(i) = (\tau(i - 1) + P) \mod(M) \quad 1 \leq i \leq M - 1. \]

\[ \tau(0) = 0 \]  \hspace{1cm} (3)

where \( P(<M) \) is the prime number and the \( P \) is not divided exactly by the \( M \).