8 Analysis of LTV Systems

8.1 Introduction

Many engineering problems need a time-varying modeling and some examples are shown in this section. The most common class of LTV systems is the one of periodic systems. They are encountered in signal processing and communications as, for example, filters which incorporate modulators in the signal path (see, e.g., [252] in which spectral characterizations of linear periodically time-varying systems are used to the study of gated phased-locked loops), as well as in control.

However, the physical time-dependance of some parameters is not the only reason for studying LTV systems. Indeed, LTV representations are obtained when simplifying complicated models as it is the case of the linearization of the nonlinear dynamics around a given trajectory. In [255] LTV models are used for the linear analysis of electrical circuits while [130] shows that a simplified observer design can be carried out using an LTV model.

Linear Parameter-Varying (LPV) systems form a special class of LTV systems in which the coefficients of the system depend on a time-varying parameter. LPV systems arise in the gain scheduling ([310], [212]). Again, despite the supplementary difficulty introduced by the time-dependance of the parameters, this kind of modeling can lead to important simplifications of the problem. In [287], the LPV modeling applied to the voltage control of the power systems leads to a decentralized control scheme. Other industrial LPV applications are reported in [15] and [365].

We continue here by presenting three industrial applications for which the time-varying modeling is mandatory. It is also shown how the algebraic approach presented in Part II is used to analyse the structural properties of some industrial systems.
8.2 Need of Time-Varying Models

8.2.1 Current-Mode Control of a Converter

As power electronics has been increasingly used in the control of electrical drives in the last decades, modeling of current-mode controlled converters has been a topic of interest for the control and power electronics communities.

A boost converter as considered in [338] is shown in Fig. 8.1 where \( D \) defines the impulse modulation.

![Fig. 8.1 Example boost converter](Image)

Under normal operating conditions, the switch is turned on every \( T \) seconds, and is turned off when the inductor current \( i_L(t) \) reaches a peak value of the control signal \( i_p(t) \) minus a compensation ramp. For the control design, the relation between a perturbation \( \delta i_p(t) \) in the control signal and the resulting perturbation \( \delta i_L(t) \) is investigated. The modeling usually adopted (see, e.g., [293], [338]) is based on the assumption that the input and output voltages do not vary significantly and, as a consequence, the relation between the perturbation in control and the resulting current perturbation can be approximated by a sample-and-hold system as given in Fig. 8.2.

This leads to a linear time-invariant discrete-time model

\[
H(z) = \frac{\Delta i_L(z)}{\Delta i_p(z)} = \frac{(M_1 + M_2)z}{(M_c + M_1)z - (M_c - M_2)}
\]

(8.1)

where \( M_1, M_2 \) and \( M_c \) are the slope magnitudes of the rising inductor current, falling inductor current, and slope-compensation ramp.

The main effects not modeled by the sample-and-hold approximation are the variation in sampling time and the finite slope of the current perturbation transition.

This modeling is correct at low-frequencies. Its flaws at high frequencies were pointed out in [272] where it is shown that the system in Fig. 8.2 is time-varying for control perturbations approaching half the switching frequency.