Abstract. Chapter is devoted to dynamics of mechanical and electromechanical systems. Sections dealing with mechanical systems concern holonomic and non-holonomic objects with multiple degrees of freedom. The concept of an object represented by a system of connected material points and the concept of a rigid body and connected bodies are presented. The Lagrange’s method of dynamics formulation is thoroughly covered, starting from d’Alembert’s virtual work principle. Carefully selected examples are used to illustrate the method as well as the application of the theory. Electromechanical system’s theory is also introduced on the basis of the Lagrange’s equation method, but starting from the principle of least action for a electrically charged particle in a stationary electromagnetic field. Subsequently, the method is generalized for macroscopic systems whose operation is based on electric energy and magnetic co-energy conversion. Nonlinear systems are discussed and the concept of kinetic co-energy is explained. Energy dissipation is introduced as a negative term of the virtual work of the system, and transformation of dissipation coefficients to the terms of generalized coordinates are presented in accordance with Lagrange method. Finally a number of examples is presented concerning electromechanical systems with magnetic and electric field and also selected robotic structures.

2.1 Mechanical Systems

2.1.1 Basic Concepts

Discrete system - is a system whose position is defined by a countable number of variables. In opposition to discrete system a continuous system (or distributed parameters’ system) is defined as a system with continuously changing variables along coordinates in space. Both these concepts are a kind of idealization of real material systems.

Particle – is an idealized object that is characterized only by one parameter – mass. To define its position in a three-dimensional space (3D) three variables are necessary. This idealization is acceptable for an object whose mass focuses closely around the center of the mass. In that case its kinetic energy relative the to linear (translational) motion is strongly dominant over the kinetic energy of the rotational motion. Besides, it is possible to consider large bodies as particles
(material points) in specific circumstances, for example when they do not rotate or their rotation does not play significant role in a given consideration. This is the case in the examination of numerous astronomical problems of movement of stars and planets.

**Rigid body** – is a material object, for which one should take into account not only the total mass $M$, but also the distribution of that mass in space. It is important for the rotating objects while the kinetic energy of such movement plays an important role in a given dynamical problem. In an idealized way a rigid body can be considered as a set of particles, of which each has an specific mass $m_i$, such that $\Sigma m_i = M$. Formally, the body is rigid if the distances $d_{ij}$ between particles $i,j$ are constant. The physical parameters that characterize a rigid body from the mechanical point of view are the total mass $M$, and a symmetrical matrix of the dimension 3, called the matrix of inertia. This matrix accounts for moments of inertia on its diagonal and deviation moments, which characterize the distribution of $m_i$ masses within the rigid body in a Cartesian coordinate system.

**Constraints** – are physical limitations on the motion of a system, which restrict the freedom of the motion of that system. The term system used here denotes a particle, set of connected particles, a rigid body or connected bodies as well as other mechanical structures. These limitations defined as constraints are diverse: they can restrict the position of a system, the velocity of a system as well as the kind of motion. They can be constant, time dependent or specific only within a limited sub-space. Formally, the constraints should be defined in an analytical form to enable their use in mathematical models and computer simulations of motion. Hence, they are denoted in the algebraic form as equations or inequalities.

**Cartesian coordinate system** - is the basic, commonly used coordinate system, which in a three dimensional space (3D) introduces three perpendicular straight axes. On these axes it is possible to measure the actual position of a given particle in an unambiguous way using three real numbers.

**Position** of the particle $P_i$ in that system is given by a three dimensional vector, so called radius-vector $\mathbf{r}_i$:

$$\mathbf{r}_i = r_i(x_i, y_i, z_i)$$  \hspace{1cm} (2.1)

Its coordinates are, respectively: $x_i, y_i, z_i$; see Fig. 2.1.

In complex mechanical systems, consisting of a number of particles: $i=1,2,\ldots,N$ the generalized position vector for the whole system is defined as follows:

$$\mathbf{r} = (\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N) = (x_1, y_1, z_1, x_2, y_2, z_2, \ldots, x_{3N}, y_N, z_{3N})$$

which is placed in an abstract $3N$ space. For a more convenient operation of this kind of notation of the system’s position, especially in application in various summation formulae, a uniform Greek letter $\xi_j$ is introduced:

$$\mathbf{r}_i = (\xi_{3i-2}, \xi_{3i-1}, \xi_{3i})$$

As a result of above, the position vector for the whole system of particles takes the following form: