The pressure distribution around a particle moving in a continuum is nonuniform. Integrating the pressure distribution over the surface one obtains a resulting force that is different from zero. As shown in Vol. I, Chapter 6.2, the different spatial components of the integral correspond to different forces: drag, lift and virtual mass forces. The averaging procedure over a family of particles gives some averaged forces which can be used in the computational analysis based on coarse meshes in the space. The purpose of this section is to summarize the empirical information for computation of the drag, lift and virtual mass forces in multiphase flow analysis.

2.1 Drag forces

2.1.1 Introduction

Consider the discrete velocity field \(d\) surrounded by a continuum denoted by \(c\). The force acting on a single particle multiplied by the number of particles per unit volume is

\[
f_{d} = -\frac{\alpha_d}{\pi D_d^3/6} e_{cd} \frac{1}{2} \rho_{cd} |\Delta V_{cd}| \Delta V_{cd} \frac{\pi D_d^3}{4} = -\alpha_d \rho_{cd} \frac{1}{D_d} \frac{3}{4} e_{cd} |\Delta V_{cd}| \Delta V_{cd}.
\]

(2.1)

Next we describe some experimentally observed effects influencing the drag coefficient \(e_{cd}\).

It is well known from experimental observations that the drag coefficient depends on the radius, on the particle Reynolds number based on the absolute value of the relative velocity, and on whether the particle is a solid, bubble, drop of pure liquid, or drop of liquid with impurities of microscopic solid particles. Because of internal circulation in the liquid or gas particle, its drag coefficient can be 1/3 of the corresponding drag coefficient of a solid particle with the same radius and Reynolds number. Small impurities hinder the internal circulation and lead to drag coefficients characteristic for solid particles.

Drag coefficients on deformed particles are 2- to 3-times larger than rigid sphere drag coefficients.

During the relative motion each particle deforms the continuum. Increasing the particle concentration leads to increased resistance to the deformation in a
restricted geometry. This means that the mechanical cohesion of the family of particles with the surrounding continuum is stronger than the cohesion of the single particle moving with the same relative velocity. In other words, the drag coefficient of a single particle with a given radius and Reynolds number is less than the drag coefficient of a particular particle with the same radius and Reynolds number, belonging to a family of particles collectively moved through the continuum.

From practical observations it seems that this phenomenon takes place up to a given particle size and then inverses for larger particle sizes in the swarm.

The velocity $V_{\text{wake}}$ at any location $\delta$ behind a solid particle with diameter $D_d$, caused by its wake, for example

$$V_{\text{wake}} \approx \Delta V_{cd} \left[ 0.2 + 0.24 \delta / D_d + 0.04 \left( \delta / D_d \right)^2 \right], \quad (2.2)$$

Stuhmiller et al. (1989), can influence the interaction of the following particle with the continuum. For one and the same concentration and form of the particles, the less the compressibility of the particle, the greater the drag coefficient.

### 2.1.2 Drag coefficient for single bubble

#### 2.1.2.1 Bubble deformation not considered

Next we will consider four regimes of mechanical interaction of flow with a single spherical bubble.

Hadamard (1911) and Rybczynski (1911) analyzing flow of a viscous fluid 2 around a sphere 1 having other viscosity for $Re \ll 1$ succeeded in finding analytical stream functions inside and outside the sphere and computed a drag coefficient

$$c_{21}^d = \frac{3\eta^* + 2}{\eta^* + 1} \frac{8}{Re}, \quad (2.3)$$

depending on the viscosity ratio $\eta^* = \eta_1 / \eta_2$ and Reynolds number for a single bubble,

$$Re = D_d \rho_2 \left| \Delta V_{12} \right| / \eta_2. \quad (2.4)$$

For infinite viscosity of the internal fluid, which is equivalent to a solid sphere, the solution reduces to the famous Stokes solution $c_{21}^d = 24 / Re$. For an inviscid bubble $c_{21}^d = 16 / Re$. As already mentioned, bubbles in liquids with impurities behave as solid spheres.

1) For low relative velocities

$$Re \leq 16, \quad (2.5)$$