4 On the variety of notations of the energy conservation for single-phase flow

The Greek word τροπη stems from the verb εντροπειη – conversion, transformation, and was used by R. Clausius in a sense of “value representing the sum of all transformations necessary to bring each body or system of bodies to their present state” (1854 Ann. Phys., 125, p. 390; 1868 Phil. Mag., 35, p. 419; 1875 “Die mechanische Wärmetheorie”, vol. 1).

This chapter recalls the achievements of the classical thermodynamics for describing the thermodynamic behavior of flows. At the end of the 19th century this knowledge had already formed the classical technical thermodynamics.

The classical thermodynamics makes use of different mathematical notation of the first principles. The purpose of this chapter is to remember that all mathematically correct transformations from one notation into the other are simply logical and consistent reflections of the same physics. The use of the specific entropy is not an exception. The importance of the entropy is that it allows one to reflect nature in the simplest mathematical way.

Even being known since 100 years the basic system of partial differential equations is still not analytically solved for the general case. The requirements to solve this system numerically and the virtual possibility to do this by using modern computers is the charming characteristic of the science today. The obtained numerical results have always to be critically examined. As we demonstrate here for pressure wave analysis lower order numerical methods predict acceptably only the first few cycles but then the solution degrades destroyed by numerical diffusion. The inability of the lower order numerical methods to solve fluid mechanics equations for initial and boundary conditions defining strong gradients is not evidence that the equations are wrong. That high accuracy solution methods are necessary to satisfy future needs of the industry is beyond question.

4.1 Introduction

Chapter 4 is intended to serve as an introduction to Chapter 5. The computational fluid mechanics produces a large number of publications in which the mathematical notation of the basic principles and of the thermodynamic relationships is often taken by the authors for self understanding and is rarely
explained in detail. One of the prominent examples is the different notation of the energy conservation for flows in the literature. The vector of the dependent variables frequently used may contain either specific energy, or specific enthalpy, or specific entropy, or temperature etc. among other variables. If doing the transformation from one vector into the other correctly the obtained systems of partial differential equation must be completely equivalent to each other. Nevertheless, sometimes the message of the different notations of the first principle is misunderstood by interpreting one of them as “wrong” and other as right. That is why recalling the basics once more seems to be of practical use in order to avoid misunderstandings.

4.2 Mass and momentum conservation, energy conservation

The fluid mechanics emerges as a scientific discipline by first introducing the idea of control volume. The flowing continuum can be described mathematically by abstracting a control volume inside the flow and describing what happens in this control volume. If the control volume is stationary to the frame of reference the resulting description of the flow is frequently called Eulerian description. If the control volume is stationary to the coordinate system following the trajectory of fluid particles the description is called Lagrangian. Oswatitsch’s “Gasdynamik” (1952) is my favorite recommendation to start with this topic because of its uncomplicated introduction into the “language of the gas dynamics”. Next we write a system of quasi linear partial differential equations describing the behavior of a single-phase flow in an Eulerian control volume without derivation.

The idea of conservation of matter was already expressed in ancient philosophy. Much later, in 1748, M. Lomonosov wrote in a letter to L. Euler “... all changes in the nature happens so that the mass lost by one body is added to other ...”. Equation (4.1) reflects the principle of conservation of mass per unit volume of the flow probably first mathematically expressed in its present form by D’Alembert (1743). The second equation is nothing else than the first principle of Newton (1872) applied on a continuum passing through the control volume probably for the first time by Euler (1737-1755), see Euler (1757). Again the equation is written per unit flow volume. The third equation reflects the principle of the conservation of energy, Mayer (1841-1851), see Mayer (1757), a medical doctor finding that the humans loose more energy in colder regions in the globe then in hotter, which is reflected in the color of the blood due to the different degree of blood oxidation.

\[
\frac{\partial p}{\partial \tau} + \nabla \cdot (\rho \mathbf{V}) = 0 \tag{4.1}
\]

\[
\rho \left( \frac{\partial \mathbf{V}}{\partial \tau} + \mathbf{V} \nabla \cdot \mathbf{V} \right) + \nabla \cdot \mathbf{p} + \rho \mathbf{g} = 0 \tag{4.2}
\]