CHAPTER 20

Credit, Credit Derivatives, and Credit Default

20.1 Introduction

After some empirics we want to build up some theory. This will be done in this and the next chapter. As shown in ch. 19, one of the accelerating and magnifying forces in the recent financially driven boom-bust cycle seems to have come from credit and credit derivatives. Before going into the securitization of debt instruments, which played an important role in the financial meltdown of the years 2007-9, we want to discuss some basics of credit derivatives. The securitization of debt instruments will be discussed in ch. 21. In this chapter we will pursue what has been called the value based approach to credit and credit derivatives.

Let us first note that for a long time the efficient market theory was a major approach in modern finance. It also underlies the Modigliani-Miller Theorem (1958) and the credit derivative and bond pricing approach by Merton (1973, 1974), both using the adding up theorem where it is assumed that $V = B + E$ (value of the firm = debt + equity). We have already contrasted the theory of efficient markets with other theories in chs. 4 and 5. We have shown there that asymmetric information, moral hazard and adverse selection become a relevant issue for realistically studying borrowing and lending. Subsequently, we have discussed intertemporal models of borrowing and lending and made statements when debt is sustainable and when a credit or debt crisis may occur: if debt is not sustainable the economic units eventually become insolvent.

Another reason for insolvency may be a lack of liquidity of economic agents. This issue has been discussed and studied in chs. 4 and 5 as well. If banks and firms go heavily into short term debt to finance long term positions, they make themselves vulnerable against short term liquidity shocks. If the financial market cuts off firms and banks from short term borrowing, they face a liquidity crisis that sometimes also leads to insolvency and bankruptcy.

Yet what we mainly discuss here is not a liquidity crisis but rather an insolvency crisis. We elaborate in a basic manner on credit derivatives as derived from Merton (1974) using Brownian motions as determining the value of the underlying assets. Critical debt and credit crisis is then discussed in the framework of the modern KMV Model. Most of our elaborations can be easily applied to households, firms, banks, real estate, governments, and countries.


## 20.2 A Model of Credit and Asset Accumulation

A more realistic theory of capital markets suggests practical rules on how to deal with credit risk and the loss of creditworthiness. Certain rules from credit contracts are typically imposed on borrowers.

When economic agents borrow, they have to pay an interest rate. In general one could make the interest payment endogenous, by allowing risk premia to be paid depending on the agents’ risk characteristics, for example the credit cost to be paid could depend on the net worth of the economic agents. One could then introduce an equation for the evolution of debt such as

\[
\dot{B} = \theta(B, k)B - S_t
\]  

(20.1)

wherein \(\theta(B, k)\) is the risk premium and \(S_t\) the net income flow of the agent. In the finance literature this risk or default premium is seen to be caused by both high leverage of the agent (for example a household or firm) as well as high volatility of its asset value, see Merton (1974). Both determine the distance-to-default, see later the KMV model.

In Grün and Semmler (2005) the asset value, the value of a firm for example, is derived from the intertemporal behavior of firms for the deterministic case. In Grün, Semmler and Bernard (2007) the stochastic case is also considered.

For the intertemporal model of lending the transversality condition should hold

\[
\lim_{t \to \infty} e^{-\theta t} B(t) = 0.
\]  

(20.2)

The ability of an obligator to service the debt, i.e. the feasibility of a contract, will depend on the obligator’s source of income. Along the lines of intertemporal models of borrowing and lending\textsuperscript{215}, we model this source of income as arising from a stock of capital \(k(t)\), at time \(t\), which changes with the investment rate \(j(t)\) at time \(t\) through

\[
\dot{k}(t) = j(t) - \sigma k(t), \quad k(0) = k_0.
\]  

(20.3)

In our general model both the capital stock and the investment are allowed to be multivariate, but here we will use one state variable for the capital stock only. The capital stock generates a net income which can be defined as:

\[
f(k, j) = k^\alpha - j - j^\beta k^{-\gamma} - c
\]  

(20.4)

This accounts for adjustment cost of capital , with \(\beta > 0, \alpha > 0, \gamma > 0\) being constants.\textsuperscript{216}

\textsuperscript{215} Prototype models used as basis for our further presentation can be found in Blanchard (1983) and Blanchard and Fischer (1989).

\textsuperscript{216} Note that the production function \(k^\alpha\) may have to be multiplied by a scaling factor. For the analytics we leave it aside here.