Chapter 6
Financial Risks and Their Modelling

The aim of this chapter is to educate the readers, in order that they understand the basics of financial risk management and so that they can interpret the numbers within this report. For the underlying abstract valuation concept we refer to appendix C.

6.1 The Model Underlying Financial Risks

In order to develop a model for managing and measuring financial risks we have a look at the balance sheet, which have seen earlier in this book:
It is clear that we need to decouple the valuation $\pi_t$ from the underlying asset. So formally the balance sheet consists of assets $(A_i)_{i \in S_A}$ and Liabilities $(L_i)_{i \in S_L}$ and we assume that both index sets $S_A$ and $S_L$ are finite. Now assume we have 1000 shares from HSBC. We could say that these 1000 shares are “one” asset. On the other hand we could model the same holding as holding 1000 pieces of the asset “1 HSBC share”. Therefore we denote by $(\alpha_i)_{i \in S_A}$ and $(\lambda_i)_{i \in S_L}$ the number of units which we own at the certain point of time. Furthermore we want to separate the shareholder equity from the liabilities and we denote it $E$.

If we write $\alpha_1 A_1$ we assume that we are holding $\alpha_1$ units of the asset $A_1$. Hence our portfolio is an abstract finite dimensional linear vector space $\mathcal{Y} = \text{span}\{(A_i)_{i \in S_A}, (L_i)_{i \in S_L}, E\}$. In this context our balance sheet is a point $x = \sum_{i \in S_A} \alpha_i A_i + \sum_{i \in S_L} \lambda_i L_i \in \mathcal{Y}$.

As seen before some assets and liabilities can be further decomposed in simpler assets and liabilities and hence we can find a suitable basis for the vector space $\mathcal{Y} = \text{span}\{e_1, \ldots, e_n\}$, where $(e_k)_{k \in \mathbb{N}_n}$ is its basis, and we remark that we can also write our balance sheet as $x = \sum_{k \in \mathbb{N}_n} \gamma_k e_k$.

The idea to introduce $\mathcal{Y}$ is to have a normalised vector space. Assume for example that we hold some ordinary bonds. In this case we would use as $e_k = Z_{(k)}$, the corresponding zero coupon bonds, etc.

We finally remark that the balance sheet $x \in \mathcal{Y}$ actually represents a random cash flow vector, and hence we strictly have $x_t$ or $X_t(\omega) \in \mathcal{X}$ if we assume that the changes of the portfolio follow a stochastic process (cf. appendix D). For measuring the risk of the actual balance sheet it is normally sufficient to assume that $y \in \mathcal{Y}$ does not change.

Next we need to look at the second part, namely the valuation $\pi_t$, and we remark that:

- The valuation is dependent on time.
- We assume that the valuation is a linear functional $\pi_t : \mathcal{Y} \to \mathbb{R}$ which allocates to each asset its value (see also appendix C).
- A liability $\mathcal{L}$ is characterised by $\pi(\mathcal{L}) \leq 0$. In the same sense and asset has a positive value. As a consequence an $x \in \mathcal{Y}$ can in principle be both an asset or