Chapter 9
Capital Models and Integrated Risk Management

9.1 Introduction

In this chapter we want to see how the different pieces of the capital models flow together in order to get an integrated capital model, covering all the different risk categories. In a lot of cases the capital models for insurance companies have been designed along the following categories:
• Financial and ALM risk,
• Life insurance risk,
• General insurance risk,
• Operational risk.

The reason for building these distinct risk modules was that there were people focusing on ALM issues, such as life risk etc. Hence it was a consequence of the relative skill set of the people and of the relative importance of the risks. Sometimes some of the risks were merely modelled as a consequence of a regulatory requirement, such as operational risks. The methods used for the different risk categories are often different and ultimately there is the question on how to link the sub-modules together. This is the same question as linking the individual risk factors within each risk module together.

From a holistic point of view it is important that the company can cover the required capital stemming from all risk types with the available risk capital.

9.2 Bringing the Puzzle Together

From a technical point of view we are in a situation where we have a set of risk categories \((\mathcal{R}_\kappa)_{\kappa \in \mathcal{K}}\) and for each risk category \(\kappa \in \mathcal{K}\) we have a corresponding loss function \(X_\kappa\) with a probability density function \(F_{X_\kappa}(t)\). So what we actually know per \(X_\kappa\) is its marginal distribution if we consider \((X_\kappa)_{\kappa \in \mathcal{K}}\) as a multidimensional random variable.

One possibility is to link the \((\mathcal{R}_\kappa)_{\kappa \in \mathcal{K}}\) together with copulas. In order to understand this concept, we need to look at two random variables \(X\) and \(Y\) with cumulative distribution functions \(F_X\) and \(F_Y\) respectively. Furthermore we remark that in this case both of the following random variables \(\tilde{X}\) and \(\tilde{Y}\) are uniformly \([0, 1]\)–distributed:

\[
\tilde{X} = F_X(X), \\
\tilde{Y} = F_Y(Y).
\]

We remark that if \((X, Y)\) are dependent, this holds true also for \((\tilde{X}, \tilde{Y})\). A copula is hence a function which transforms the random variables \((\tilde{X}, \tilde{Y})\). More formally: A copula is a multivariate (n-dimensional) joint distribution on \([0, 1]^n\), such that every marginal distribution is uniform on the interval \([0, 1]\). A function

\[
C : [0, 1]^n \rightarrow [0, 1], (x_1, \ldots, x_n) \mapsto C(x_1, \ldots, x_n)
\]

is an n–dimensional copula if the following hold: