5 Influence of the interfacial forces on the turbulence structure

The interfacial forces are in interaction with all other forces in the momentum equations. Experiments for investigation of the turbulence are concentrated always on some collective action of the forces. Therefore they have to be described correctly. Of course, performing simple analytical experiments by isolating only one force is what is always needed but very difficult. We discuss in this chapter the role of some of the forces which are still under investigation world wide.

5.1 Drag forces

In Kolev (2007b) constitutive relation for computation of the interfacial drag forces for variety of flow pattern and regimes are available. In the previous chapter we saw clearly that the drag forces contribute to the generation of turbulence in the trace of the bubbles. So using these two references the effect of the drag forces can be calculated.

5.2 The role of the lift force in turbulent flows

Note on particle rotation: A rotating sphere obeys the law

\[ I_d \frac{d\omega_d}{d\tau} = -C_d^\omega \left( \frac{D_d}{2} \right)^5 \frac{1}{2} \rho_d |\omega_d| |\omega_d| \cdot \]  \hspace{1cm} (5.1)

Here the particle rotation velocity is \( \omega_d \) and \( I_d \) is the particle’s moment of inertia.

\[ C_d^\omega = \frac{c_1}{\left( \text{Re}_{cd}^{\omega} \right)^{1/2}} + \frac{c_2}{\text{Re}_{cd}^{\omega}} + c_3 \text{Re}_{cd}^{\omega} \]  \hspace{1cm} (5.2)

is a coefficient depending on the rotation Reynolds number.
\[ \text{Re}^{\infty}_{cd} = \left( \frac{D}{2} \right) \frac{n}{V_c}. \]  

(5.3)

The c-coefficients are given by Yamamoto et al. (2001) in the following table:

<table>
<thead>
<tr>
<th>( \text{Re}^{\infty}_{cd} )</th>
<th>0 to 1</th>
<th>1 to 10</th>
<th>10 to 20</th>
<th>20 to 50</th>
<th>&gt;50</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_1 )</td>
<td>0</td>
<td>0</td>
<td>5.32</td>
<td>6.44</td>
<td>6.45</td>
</tr>
<tr>
<td>( c_2 )</td>
<td>50.27</td>
<td>50.27</td>
<td>37.2</td>
<td>32.2</td>
<td>32.1</td>
</tr>
<tr>
<td>( c_3 )</td>
<td>0</td>
<td>0.0418</td>
<td>5.32</td>
<td>6.44</td>
<td>6.45</td>
</tr>
</tbody>
</table>

We learn from this dependence that both small and light particles can be easier to rotation compared to heavy and large particles. The following three main idealizations gives an idea for origination of the so-called lift force:

a) Rotating symmetric particle in symmetric flow of continuum experiences a lift force called Magnus force (a Berlin physicist Gustav Magnus 1802-1870). The curiosity of Lord Rayleigh to explain the trajectory of the tennis ball lead him in 1877 to the corresponding explanation. The force was analytically estimated by Jukowski and independently by Kutta, see in Albring (1970) p. 75.

b) Non-rotating symmetric particle in non-symmetric continuum flow experiences lift force, Jukowski.

c) Non-rotating asymmetric particle in symmetric continuum flow experiences lift force, Jukowski.

A combination of the radial liquid and gas momentum equations results to

\[ p(r) = p(R) - (1 - \alpha_r) \rho v^2 - \int_r^R (1 - \alpha_r) \rho \left( \frac{u^2 - v^2}{r^*} \right) dr^*. \]  

(5.4)

The above equation reduces to the Eq. 4a in Laufer (1953) for zero-void. Later it was found that this is valid for bubbles with small sizes. If fluctuation velocities in the radial and in the azimuthally directions, respectively, are obtained from experiment and the void profile, the pressure variation along the pipe radius can be computed. If it fits to the measured, there is no need for other forces to explain the physics. But if there are differences, they may come from the so-called lift, lubrication and dispersion forces. The steady state momentum equations in radial direction for bubbles and liquid are

\[ \alpha_r \frac{dp}{dr} - \frac{1}{r} \frac{d}{dr} \left( \alpha_r r \tau_{rr} \right) + \frac{1}{r} \alpha_r \tau_{r,\theta} - f_{r,21}^L = 0, \]  

(5.5)