Chapter 1
Classical Theories of Plasticity

The macromechanical theories (also called the mathematical theories) of plasticity describe plastic deformations phenomenologically, on the macroscopic level, and establish relations among macroscopic mechanical quantities (such as stresses and strains). These theories are based on the notions of a yield surface and a hardening rule, governing the evolution of the yield surface (loading surface) and the stress-strain relations of the material. The analysis of classical theories of plasticity and their concurrence with experimental data are the focus of this chapter.

The classical theories considered here are based upon the following assumptions: (i) only small plastic strains are considered; (ii) the material is initially isotropic until an inelastic behavior occurs; (iii) work-hardening materials (except for Section 1.3 and 1.14 with perfectly plastic materials) are considered; (iv) the plastic strain involves only a change in shape but no change in volume; (v) materials behave in tension in a same manner to their behavior in compression; and (vi) temporary effects, influences of temperature and other actions (irradiation, magnetic field etc) are neglected [17,18,53,125,126].

1.1 Definition of Subject

Plasticity is a property of a material to undergo a non-reversible change of shape in response to an applied force. The theory of plasticity, being the section of continuum mechanics, is concerned with the analysis of stresses and elasto-plastic strains in a body.

A start point of classical theories of plasticity is the experimental macro-behavior of specimens during loading. Typical stress-strain curve of polycrystalline body has the form schematically shown in Fig. 1.1a. Such curve can be obtained from a tensile test, or at inner pressure of short cylinders as well as at torsion of thin-walled shafts.

Figure 1.1a shows the typical stress-strain curve for a simple tension specimen. One can point out characteristic points in this diagram. The proportional limit of a material is defined as a largest value of stress for which the stress is still proportional to the strain. The elastic limit of a material is defined as a largest value of stress that can be applied without causing any residual strain upon removal for the stress. If the material is unloaded from point $M$, beyond the elastic
limit, and then loaded again, the unloading-loading portion $MO'M'$, hysteresis loop, appears as in Fig. 1.1a.

Fig. 1.1 Real (a) and idealized (b) stress-strain diagram.

In terms of classical theories, real diagrams are replaced by idealized ones (Fig. 1.1b) where there is only one characteristic point, yield strength (yield limit) of the material, $\sigma_s$, which is the stress required to produce a certain residual plastic strain, usually 0.2%. For $\sigma < \sigma_s$, the loading portion is assumed to obey Hooke’s law. Furthermore, the unloading $MO'$ and the subsequent loading