3.1 Introductory Remarks

A new theory, the synthetic theory of plasticity, is presented in this chapter. It combines the Sanders flow plasticity theory and the Batdorf-Budiansky concept of slip.

We demand that the synthetic theory should satisfy the following requirements:

(i) As it was repeatedly noted, the simplest theory of plasticity, the Hencky-Nadai deformation theory, shows satisfactory agreements with experiments in proportional (simple) loading for many materials. Therefore, it is logical to require that stress-strain relationships of the synthetic theory reduce to the deviator proportionality at simple loading.

(ii) The arising of corner point on loading surface; it appears to be the only way to model plastic strain increments in orthogonal additional loading.

(iii) The two-level determination of plastic strain; as shown earlier two-level models are more effective than one-level ones (see Sec. 2.7 points 1 and 2, and Sec. 2.14 points 1-3).

It must be noted that the synthetic theory is not applicable to an arbitrary state of stress, the maximal number of non-zero stress tensor components is four: all three normal and single shear stress tensor components (other two shear stress components are equal to zero).

The bases of synthetic theory have been elaborated in the framework of the J. Andrusik candidate dissertation under K. Rusinko’s supervision.

3.2 Partial Cases of the Tresca Yield Surface

Since the Batdorf–Budiansky concept of slip is based on the Tresca yield criterion, let us consider in more detail the construction of the Tresca yield surface. Consider the plane stress state ($\sigma_x \neq 0, \sigma_z \neq 0, \tau_{xz} \neq 0$) and the case when $\sigma_x \neq 0, \sigma_z \neq 0, \tau_{xy} \neq 0$. According to the Tresca yield criterion, yielding begins when the shear stress reaches a critical value, $\sigma_S / 2$.:
\[ \sigma_{\text{max}} - \sigma_{\text{min}} = \sigma_S, \]  

(3.2.1)

where \( \sigma_{\text{max}} \) and \( \sigma_{\text{min}} \) are the maximal and minimal principal stresses acting in an element of a body; they are determined by well-known cubic equation. For the plane stress system the principal stresses are determined by Eq. (2.7.1). Consider the following three variants

a) \( \sigma_1 > \sigma_2 > 0, \sigma_3 = 0, \sigma_{\text{max}} = \sigma_1, \sigma_{\text{min}} = 0. \) Then from Eq. (3.2.1) it follows that \( \sigma_1 = \sigma_S, \) i.e.,

\[ \tau_{xz}^2 = \sigma_x \sigma_z - \sigma_S (\sigma_x + \sigma_z) + \sigma_S^2. \]

(3.2.2)

This is the equation of cone whose apex is at the point with coordinates \( \sigma_x = \sigma_z = \sigma_S \) and \( \tau_{xz} = 0 \), and its axis coincides with bisector \( \sigma_x = \sigma_z \) of \( \tau_{xz} = 0 \)-plane.

b) \( \sigma_1 > 0, \sigma_2 < 0, \sigma_3 = 0, \sigma_{\text{max}} = \sigma_1, \sigma_{\text{min}} = \sigma_2. \) Now, the yield condition, \( \sigma_1 - \sigma_2 = \sigma_S \), is

\[ (\sigma_x - \sigma_z)^2 + 4\tau_{xz}^2 = \sigma_S^2. \]

(3.2.3)

This is the equation of elliptic cylinder whose axis is the bisector \( \sigma_x = \sigma_z \) of \( \tau_{xz} = 0 \)-plane; the semi-major and semi-minor axes of the base-ellipse are \( \sigma_S/\sqrt{2} \) and \( \sigma_S/2 \), respectively.

c) \( \sigma_2 < \sigma_1 < 0, \sigma_3 = 0, \sigma_{\text{max}} = 0, \sigma_{\text{min}} = \sigma_2. \) Then \( -\sigma_2 = \sigma_S, \) i.e.,

\[ \tau_{xz}^2 = \sigma_x \sigma_z + \sigma_S (\sigma_x + \sigma_z) + \sigma_S^2 \]

(3.2.4)

giving a cone with apex at the point \( \sigma_x = \sigma_z = -\sigma_S \) and \( \tau_{xz} = 0 \), the cone axis is the same as in (3.2.2).

The yield surface constructed in \( \sigma_x - \sigma_z - \tau_{xz} \) stress space is shown in Fig. 3.1, portions 1, 2, and 3 are constructed on the base of Eqs. (3.2.4), (3.2.3), and (3.2.2), respectively. The surface consists of the elliptic cylinder 2, bounded by the cones 1 and 3. The intersection of the surface and \( \sigma_x = 0 \)-plane gives the line given by Eq. (2.6.12). The projection of the surface on \( \tau_{xz} = 0 \)-plane gives the Tresca hexagon.