The Mutex Paradigm of Concurrency

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Abstract. Concurrency can be studied at different yet consistent levels of abstraction: from individual behavioural observations, to more abstract concurrent histories which can be represented by causality structures capturing intrinsic, invariant dependencies between executed actions, to system level devices such as Petri nets or process algebra expressions. Histories can then be understood as sets of closely related observations (here step sequences of executed actions). Depending on the nature of the observed relationships between executed actions involved in a single concurrent history, one may identify different concurrency paradigms underpinned by different kinds of causality structures (e.g., the true concurrency paradigm is underpinned by causal partial orders with each history comprising all step sequences consistent with some causal partial order). For some paradigms there exist closely matching system models such as elementary net systems (EN-systems) for the true concurrency paradigm, or elementary net systems with inhibitor arcs (ENI-systems) for a paradigm where simultaneity of executed actions does not imply their unorderedness.

In this paper, we develop a system model fitting the least restrictive concurrency paradigm and its associated causality structures. To this end, we introduce ENI-systems with mutex arcs (ENIM-systems). Each mutex arc relates two transitions which cannot be executed simultaneously, but can be executed in any order. To link ENIM-systems with causality structures we develop a notion of process following a generic approach (semantical framework) which includes a method to generate causality structures from the new class of processes.

Keywords: concurrency paradigms, elementary net systems, inhibitor arcs, mutex arcs, semantical framework, step sequences, process and causality semantics.

1 Introduction

Concurrency can be studied at different levels of abstraction, from the lowest level dealing with individual behavioural runs (observations), to the intermediate level of more abstract concurrent histories which can be represented by causality structures (or order structures) capturing intrinsic (invariant) dependencies between executed actions, to the highest system level dealing with devices such as Petri nets or process algebra expressions. Clearly, different descriptions of concurrent systems and their behaviours at these distinct levels of abstractions must be consistent and their mutual relationships well understood.
Abstract concurrent histories can be understood as sets of closely related observations. In this paper, each observation will be a step sequence (or stratified poset) of executed actions. For example, Figure 1(a) depicts an EN-system generating three step sequences involving the executions of transitions a, b and c, viz. \( \sigma_1 = \{a, b\}\{c\} \), \( \sigma_2 = \{a\}\{b\}\{c\} \) and \( \sigma_3 = \{b\}\{a\}\{c\} \). They can be seen as belonging to a single abstract history \( \Delta_1 = \{\sigma_1, \sigma_2, \sigma_3\} \) underpinned by a causal partial order in which a and b are unordered and they both precede c. From our point of view it is also important to note that \( \Delta_1 \) adheres to the true concurrency paradigm captured by the following general statement:

Given two executed actions (e.g., a and b in \( \Delta_1 \)), they can be observed as simultaneous (e.g., in \( \sigma_1 \)) \( \iff \) they can be observed in both orders (e.g., a before b in \( \sigma_2 \), and b before a in \( \sigma_3 \)). (TRUECON)

Concurrent histories adhering to such a paradigm are underpinned by causal partial orders, in the sense that each history comprises all step sequences consistent with some causal partial order on executed actions. Elementary net systems \([18]\) (EN-systems) provide a fundamental and natural system level model for the true concurrency paradigm. A suitable link between an EN-system and histories like \( \Delta_1 \) can be formalised using the notion of a process or occurrence net \([118]\). Full consistency between the three levels of abstraction can then be established within a generic approach (the semantical framework of \([14]\)) aimed at fitting together systems (nets from a certain class of Petri nets), abstract histories and individual observations.

Depending on the exact nature of relationships holding for actions executed in a single concurrent history, similar to (TRUECON) recalled above, \([9]\) identified eight general concurrency paradigms, \( \pi_1 - \pi_8 \), with true concurrency being another name for \( \pi_8 \). Another paradigm is \( \pi_3 \) characterised by (TRUECON) with \( \iff \) replaced by \( \Leftarrow \). This paradigm has a natural system level counterpart provided by elementary net systems with inhibitor arcs (ENI-systems). Note that inhibitor arcs (as well as activator arcs used later in this paper) are well suited to model situations involving testing for a specific condition, rather than producing and consuming resources, and proved to be useful in areas such as communication protocols \([2]\), performance analysis \([4]\) and concurrent programming \([5]\).

![Fig. 1. EN-system (a); ENI-system with an inhibitor arc joining the output place of transition b with transition a implying that a cannot be fired if the output place of b is not empty (b); and ENIM-system with a mutex arc between transitions a and b implying that the two transitions cannot be fired in the same step (c)](image-url)