Frequency Capping in Online Advertising  
(Extended Abstract)

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Abstract. We study the following online problem. Each advertiser $a_i$ has a value $v_i$, demand $d_i$, and frequency cap $f_i$. Supply units arrive online, each one associated with a user. Each advertiser can be assigned at most $d_i$ units in all, and at most $f_i$ units from the same user. The goal is to design an online allocation algorithm maximizing total value.

We first show a deterministic upper bound of $3/4$-competitiveness, even when all frequency caps are 1, and all advertisers share identical values and demands. A competitive ratio approaching $1 - 1/e$ can be achieved via a reduction to a model with arbitrary decreasing valuations \cite{GM07}. Our main contribution is analyzing two $3/4$-competitive greedy algorithms for the cases of equal values, and arbitrary valuations with equal demands. Finally, we give a primal-dual algorithm which may serve as a good starting point for improving upon the $1 - 1/e$ ratio.

1 Introduction

Display advertising, consisting of graphic or text-based ads embedded in webpages, constitutes a large portion of the revenue from Internet advertising, totaling billions of dollars in 2008. Display, or brand, advertising is typically sold by publishers or ad networks on a pay-per-impression basis, with the advertiser specifying the total number of impressions she wants (the demand) and the price she is willing to pay per impression\textsuperscript{1}.

Since display ads are sold on a pay-per-impression rather than on a pay-per-click or pay-per-action basis, effective delivery of display ads is very important to maximize advertiser value — each impression that an advertiser pays for must

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\textsuperscript{1} In contrast, sponsored search advertisers typically pay per click or per action, and usually have budgets, rather than demands, or quotas, on the number of impressions.
be shown to *as valuable a user* as possible. One aspect of effectively delivering display ads, which has been widely studied, is good targeting — matching ads to users who are likely to be responsive to the content of the ad. Another very important, but less studied, aspect is *limiting user exposure* to an ad - displaying the same ad to a user multiple times diminishes value to the advertiser, since the incremental benefit from repeatedly displaying the same ad to a user is likely to be small (a user is unlikely to react to an ad after he has seen it a few times).

The notion of limiting the number of times a user is exposed to a particular ad is called *frequency capping* [19], and is often cited as a way to avoid banner ad burnout. That is, frequency capping prevents ads from being displayed repeatedly to the point where visitors are being overexposed and response drops. Serving frequency capped ads is a very real requirement to maximize value delivered to display advertisers, particularly in the pay-per-impression structure of the display advertising marketplace. This is recognized by a number of publishers and ad networks (for instance, RightMedia, Google and Yahoo!) who already offer or implicitly implement frequency capping for their display advertisers.

Serving display ads subject to a frequency capping constraint poses an *online assignment* problem since the supply of users, or impressions, is not known to the ad server in advance. How should the ad server allocate impressions to advertisers in this setting? In this paper, we study the simplest abstractions of the assignment problems motivated by frequency capping.

**Problem Statement.** There are \(n\) advertisers. Advertiser \(i\) has value per impression \(v_i\), which is the price she is willing to pay for an impression, and a demand \(d_i\), which is the maximum number of impressions she is interested in. In addition, she also has a frequency cap \(f_i\), which is the maximum number of times her ad can be displayed to the same user. That is, an advertiser pays \(v_i\) only for impressions from users who have not seen her ad more than \(f_i\) times. The set of advertisers, and their parameters, is known to the ad server in advance.

Impressions from users arrive online. We say an advertiser is *eligible* for an impression if she still has leftover demand, and has not yet exhausted her frequency cap for the user associated with this impression. When an impression arrives, the ad server must immediately decide which ad, among the set of eligible advertisers, to display for that impression. The total revenue obtained by an algorithm is the sum of the revenues from each impression it allocates. We want to design algorithms that are competitive against the optimal offline allocation, which knows the supply of impressions (with their associated users) in advance. (This problem is captured by the model in [14], see \(\square\)).

In the absence of the frequency capping constraint \((f_i = \infty)\), the natural greedy algorithm, which assigns each arriving impression to the advertiser with the highest per-impression value \(v_i\), is optimal. However, with the frequency capping constraint, the ad server faces a tradeoff between assigning an arriving impression to an advertiser with high \(v_i\) but large frequency cap (since the

\[\text{\footnotesize{\textsuperscript{2}} While it might be argued that multiple displays of an ad to a user reinforces its message, repeated display without an upper limit clearly diminishes value.}\]