Chapter 7
Dynamics and Singularity of Boundary Flows

In this chapter, the switchability and attractivity of boundary flows to \((n-2)\)-dimensional edges will be discussed in order to understand edge and vertex dynamics in discontinuous dynamical systems. The basic properties of boundary flows to edges will be discussed first. The coming, leaving and tangency of a boundary flow to an \((n-2)\)-dimensional edge will be presented. The switchability and passability of a boundary flow from an accessible boundary to another accessible boundary will be presented with a switching rule. In addition, the switchability and passability of boundary flows and domain flows will be discussed with a switching rule. The equi-measuring edge for boundary flows will be introduced, and the attractivity of a boundary flow to the \((n-2)\)-dimensional edge will be presented. Finally, the bouncing characteristics of a boundary flow to the \((n-2)\)-dimensional edge will be discussed.

7.1. Boundary flow properties

In Chapter 5, the boundary flow network was discussed once flows exist only on the boundaries. To investigate the boundary flow behaviors, the basic properties of a boundary flow to the edge will be discussed. Consider an \((n-2)\)-dimensional edge to be formed by intersection of three \((n-1)\)-dimensional boundaries, as shown in Fig. 7.1. The black thick curve denotes the \((n-2)\)-dimensional edge. The investigated boundary flow is a black curve, and the referenced boundary flow is a red curve. The \((n-2)\)-dimensional edge must be intersected by two \((n-1)\)-dimensional boundary surfaces. So the other surfaces of the \((n-2)\)-dimensional edge should be correlated to the aforementioned two boundary surface. Thus, only one of the rest boundary surfaces can be adopted for the attractivity of a boundary flow to the edge. Similarly, the coming, leaving and totally grazing boundary flows to the edge are also sketched in Figs. 7.2 and 7.3.
Fig. 7.1 Boundary flows on the three \((n-1)\)-dimensional boundaries: (a) coming flow and (b) leaving flows. The dark thick curve represents the \((n-2)\)-dimensional edge. The investigated boundary flow is a black curve with arrow on the boundary. The referenced boundary flow is a gray curve with arrow. \((n_1 + n_2 + n_3 = n)\)