Optical Graph 3-Colorability

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Abstract. The graph 3-colorability problem is a decision problem in graph theory which asks if it is possible to assign a color to each vertex of a given graph using at most three colors, satisfying the condition that every two adjacent vertices have different colors. It has been proved that the graph 3-colorability problem belongs to NP-complete class of problems which no polynomial resources solution is found for them yet.

In this paper, a novel optical solution to the graph 3-colorability problem is provided. In this solution, polynomial number of black filters are created in preprocessing phase each of which has exponential size and requires exponential time to be created. After preprocessing phase, the provided solution takes $O(n + m)$ time to decide if a given graph is 3-colorable or not, where the given graph has $n$ vertices and $m$ edges.

Keywords: Unconventional Computing, Optical Computing, NP-Complete, Graph Coloring, Graph 3-Colorability.

1 Introduction

Graph coloring problems is a class of problems in graph theory seeking to assign a color to each vertex of a given graph in such a way that every two adjacent vertices have different colors. Graph coloring problems arise from many real-world applications such as scheduling [1] making it necessary to find efficient solutions for these problems.

Graph 3-colorability problem is one of the graph coloring problems having specific applications in resource allocation and scheduling [2]. The graph 3-colorability problem asks if it is possible to assign a color to each vertex of a given graph in such a way that every two adjacent vertices (two vertices which are connected via an edge) have different colors, using at most three colors. It has been proved that 3-colorability problem is an NP-complete problem and like every other NP-complete problems, no polynomial resources solution is found yet.

Light is a natural phenomenon used in computation because of its special physical properties such as its parallel motion. Many NP-complete problems have recently investigated in optical computing such as the 3-SAT problem [3], the Hamiltonian path problem [4], the exact cover problem [5], the subset sum problem [6], the maximum clique problem, the vertex cover problem, the partition
problem, the 3D-matching problem, the permanent problem, and the traveling salesman problem (TSP) [7].

New computational capabilities of optical computing in comparison to conventional computing have resulted to obtain more efficient solutions for these NP-complete problems, brings the idea that using optical computing to solve 3-colorability problem will also result to obtain more efficient solutions. Although the 3-colorability problem have been investigated in other branches of unconventional computing such as DNA computing [8] and quantum computing [9], but it seems that no optical solution for the graph 3-colorability problem (based on natural properties of light) is provided yet.

In this paper, a novel optical solution for the graph 3-colorability problem is provided. The solution takes just polynomial time to solve each problem instance, but exponential time in preprocessing phase. In the next section, the graph 3-colorability problem is defined. Provided optical solution and its complexity analysis are explained in sections 3 and 4, respectively. Finally, the conclusion of the paper is provided in section 5.

2 The Graph 3-Colorability Problem

In graph theory, graph coloring means to assign a color to each vertex of a graph. A proper graph coloring is assigning different colors to every two adjacent vertices (two vertices are adjacent if and only if they are connected via an edge). A proper 3-coloring for a given graph \( G \), is a proper coloring for \( G \) using just three colors. A given graph \( G \) is 3-colorable, if and only if there exists at least one proper 3-coloring for \( G \). Fig. 1 shows a 3-colorable graph and a proper 3-coloring solution. Fig. 2 shows a graph which is not 3-colorable.

The graph 3-colorability problem is a decision problem (requires answer “yes” or “no”) which is looking to find if a given graph \( G \) is 3-colorable or not [10]. As the graph 3-colorability problem is a decision problem, it is not seeking to find a proper 3-coloring, but just requires answer “yes” if the given graph is 3-colorable, and answer “no” otherwise.

It has been proved that the graph 3-colorability problem is an NP-complete problem [10] and the best algorithms to solve this problem is exponential time in the conventional computers.

![Fig. 1. Example of a 3-colorable graph and a possible proper 3-coloring solution. The colors assigned to the vertices are specified in the squares.](image-url)