On the Semantics of Bipolarity and Fuzziness

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Abstract. This paper analyzes the relationship between fuzziness and bipolarity, notions which were devised to address different kinds of uncertainty: linguistic imprecision, in the former, and knowledge relevance and character or polarity, in the latter. Although different types of fuzziness and bipolarity have been defined, these relations are not always clear. This paper proposes the use of four-valued extensions to provide a formal method to rigorously define and compare the semantics and logical structure of diverse combinations of fuzziness and bipolarity types. As a result, this paper claims that these notions and their different types are independent and not semantically equivalent despite its possible formal equivalence.

1 Introduction

Fuzziness [39] and bipolarity [11] are two independent but complementary notions originally (and separately) devised to face the mathematical modelling of different features of natural languages and human reasoning. Though their influence (more than considerable in fields as decision theory [14,17,26,29] or machine learning [19]) has spread separately, in the last few decades both notions have started to appear together in many developments on these and other fields (see for instance [4,6,13,15,30,31,38]), which comes to show its high relevance as a topic of research inside soft computing [20] and logics [36].

However, the relationships (and differences) between fuzziness and bipolarity are not always clear. In order to introduce our point, let us remind that, on one hand, fuzziness is concerned with the imprecision inherent to natural languages: many relevant predicates (i.e. words) $P$, as good or young, have ill-defined boundaries, and uncertainty arises regarding whether objects $x$ of a universe of discourse $X$ (e.g. decision alternatives or ages of customers) fulfil them or not.

On the other hand, bipolarity is concerned with the character (or polarity) and relevance of information: it has become clear (see [7,25,28]) that human reasoning tends to analyze reality (e.g. a decision to be taken [23,29]) by checking separately...
both the positive and negative sides of the available information (e.g. an alternative could be good for certain criteria and bad for other set of criteria) in order to acquire a more expressive and relevant knowledge. Thus, reality is judged in terms of pairs of poles of reference $P/Q$, as false/true or good/bad, which organize and give relevance to the available information.

Moreover, different types of fuzziness [40] and bipolarity [12] have been studied and defined. While usual (type-1) fuzziness (F1) measures linguistic imprecision in a precise way (assigning a gradable but precise truth-value $\mu_p(x) \in [0,1]$ to the proposition “$x$ fulfils $P$”, thus modelling $P$ as a fuzzy set), type-2 fuzziness (F2) enables such an imprecision to be measured imprecisely (since it assigns a fuzzy set of the truth scale $[0,1]$ to "$x \in P".

Similarly, while type-1 bipolarity (B1) relies on the idea that negative information is just the negation or complementation of the positive one, type-2 bipolarity (B2) allows the relation between poles to be not so simple (for example bad $\neq$ not good), and thus evaluating the pair $(\mu_p(x), \mu_Q(x))$ could be necessary in order to capture all relevant information.

Notice that, as they try to address different kinds of uncertainty, fuzziness and bipolarity seem to be not necessarily related or interlinked: in principle a B2 formalism could be either an F1 or F2 (or even crisp!) model, and an F2 framework could be associated to either a B1 or B2 setting. Nevertheless, a commonly-used instance of type-2 fuzziness, interval valued fuzzy sets (IVFS, see [18]), actually devised as B1 objects, has been shown (see [8,9]) to be in certain sense equivalent to Atanassov fuzzy sets (AFS, see [1]), which however were originally devised as F1 and B2 objects. In fact, as a consequence of this formal equivalence, a bitter dispute (see [10] and [3]) raised between Atanassov and his followers, on one side, and an important part of the fuzzy community, on the other, about the exact meaning of AFS and their real relevance in the context of bipolarity.

The main objective of this paper is to shed some light on the relations between fuzziness and bipolarity from a different perspective, and try to lead the referred differences between Atanassov and his detractors, apparently not totally solved, to a definitive solution. For this aim, the notion of four-valued extension (that clearly resembles that of preference structure) is used in order to rigorously define and compare the semantics and underlying logical structure of each possible combination of fuzziness and bipolarity types 1 and 2. This will allow us to separate and distinguish IVFS from AFS in a practical way, and will enable us to show the independency of fuzziness and bipolarity. AFS in a practical way, and will enable us to show the independency of fuzziness and bipolarity. AFS in a practical way, and will enable us to show the independency of fuzziness and bipolarity.

This paper is organized as follows: the notions of type-1 and type-2 fuzziness are revised in Section 2, and those of type-1 and type-2 bipolarity will be revised in Section 3. Four-valued extensions are introduced and applied to the four possible combinations of bipolarity and fuzziness types in Section 4. Finally, some conclusions are shed in Section 5.