6 Turbine and Compressor Cascade Flow Forces

The last chapter was dedicated to the energy transfer within turbomachinery stages. The stage mechanical energy production or consumption in turbines and compressors were treated from a unified point of view by introducing a set of dimensionless parameters. As shown in Chapter 4, the mechanical energy, and therefore the stage power, is the result of the scalar product between the moment of momentum acting on the rotor and the angular velocity. The moment of momentum in turn was brought about by the forces acting on rotor blades. The blade forces are obtained by applying the conservation equation of linear momentum to the turbine or compressor cascade under investigation. In this chapter, we first assume an inviscid flow for which we establish the relationship between the lift force and circulation. Then, we consider the viscosity effect that causes friction or drag forces on the blading.

6.1 Blade Force in an Inviscid Flow Field

Starting from a given turbine cascade with the inlet and exit flow angles shown in Fig.6.1, the blade force can be obtained by applying the linear momentum principles.
to the control volume with the unit normal vectors and the coordinate system shown in Fig. 6.1. As in Chapter 4, the blade force is:

\[ F_i = \dot{m}V_1 - \dot{m}V_2 - n_1 p_1 sh - n_2 p_2 sh \]  \hspace{1cm} (6.1)

with \( h \) as the blade height that can be assumed as unity. The relationship between the control volume normal unit vectors and the unit vectors pertaining to the coordinate system is given by: \( n_1 = -e_2 \) and \( n_2 = e_2 \). The inviscid flow force is obtained by the linear momentum equation where no shear stress terms are present:

\[ F_i = \dot{m}(V_1 - V_2) + e_2(p_1 - p_2)sh \]  \hspace{1cm} (6.2)

The subscript \( i \) refers to inviscid flow. The above velocities can be expressed in terms of circumferential as well as axial components;

\[ F_i = -e_1 \dot{m}([V_{ul} + V_{u2}] + e_2[\dot{m}(V_{ax1} - V_{ax2}) + (p_1 - p_2)sh] \]  \hspace{1cm} (6.3)

With \( V_{ax1} = V_{ax2} \) and \( V_{ul} = V_{u2} \) from Fig. 6.1, Eq. (6.3) is rearranged as:

\[ F_i = -e_1 \dot{m}(V_{u1} + V_{u2}) + e_2(p_1 - p_2)sh = e_1 F_u + e_2 F_{ax} \]  \hspace{1cm} (6.4)

with the circumferential and axial components

\[ F_u = -\dot{m}(V_{u1} + V_{u2}) \] \text{ and } \[ F_{ax} = (p_1 - p_2)sh \]  \hspace{1cm} (6.5)

The above static pressure difference is obtained from the Bernoulli equation \( p_{01} = p_{02} \), where the static pressure difference is obtained:

\[ p_1 - p_2 = \frac{1}{2} \rho (V_2^2 - V_1^2) = \frac{1}{2} \rho (V_{u2}^2 - V_{u1}^2) \]  \hspace{1cm} (6.6)

Inserting the pressure difference along with the mass flow \( \dot{m} = \rho V_{ax} sh \) into Eq. (6.5) and the blade height \( h = 1 \), we obtain the axial as well as the circumferential components of the lift force: