Dendritic Meshing*
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1 Introduction

A mesh is said to be dendritic if it contains elements with mid-side (edge) nodes when the predominant element topology has only corner nodes. A dendritic mesh is illustrated in Figure 1 where the predominant element is a four-node quadrilateral, but has also several five-node quadrilateral elements each with one mid-edge node plus four corner nodes. Such meshes arise when an approximately uniform element size is required across a mesh domain in cases, for example, where domain geometry changes would otherwise cause a significant variation in element size or in an Adaptive Mesh Refinement (AMR) context. In meshes created for multi-physics applications with explicit time-stepping, the maximum time-step size is intimately tied to element size through the Courant-Friedrichs-Lewy condition[6]:
\[
\frac{u \Delta t}{\Delta x} \leq C
\]
where \( u \) is a representative speed, \( \Delta x \) the element size, \( \Delta t \) is the time-step size, and \( C \) is a constant appropriate for the physics being modeled. This implies \( \Delta t \leq C' \Delta x \), \( C' \) a constant, and the smaller the element, the smaller the time-step size. The smallest element in a mesh therefore limits the time-step size providing motivation to equalize element size over a domain.

An additional concern is that the mesh adhere as nearly as possible to the domain boundary geometry. There are two general classes of boundary-fitted mesh generation methods (e.g., [14]): block-structured and unstructured. Both of these methods have inherent strengths and weaknesses. The various unstructured methods are highly automated and tend to produce meshes with relatively uniform zone size. However unstructured techniques

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generally produce meshes with irregular zone connectivity and there is little or no control over zone orientation or zone aspect ratio. Structured methods enable control of zone orientation, have regular connectivity, and can produce very high aspect ratio zones (desired in some problem domains). However structured techniques are labor-intensive and control of zone-size is limited by the geometry of the problem and the domain decomposition into logical blocks. The discussion herein focuses on boundary-fitted meshes with a relatively uniform zone size in most regions, the option to produce high aspect ratio zones in other regions, and the ability to align the mesh with the predominant direction of shock propagation and/or material flow. Neither of the existing meshing methods mentioned above satisfies these criteria.

To meet these meshing requirements, a hybrid method is presented which is termed dendritic meshing. Dendritic meshing is a modified block-structured technique that allows logical edges within a structured mesh block to be “deactivated” as needed to control zone size. This method combines many of the advantages of both structured and unstructured methods. With dendritic meshing, zone size can be kept relatively uniform when needed, zone aspect ratio can be controlled, zone orientation can be controlled, and nodes of irregular connectivity within the mesh are minimized.

The starting point for a dendritic mesh is a standard block-structured mesh; a mesh consisting of perhaps multiple blocks each having a logical $ij$-structure. To produce a dendritic mesh, selected segments of constant-$i$ or constant-$j$ lines are removed. The “dendrites” are the nodes at which an $i$ or $j$ line segment becomes inactive. After removing the selected mesh segments, the mesh is re-interpolated using a specialized dendrite-aware transfinite interpolation algorithm, and is then smoothed with a dendrite-aware smoother. Figures 1 and 2 show the active and removed edges of a structured mesh in physical and logical space, respectively.

The following sections highlight the significant issues in the dendritic meshing process. These issues are: data structures, feathering of a structured mesh, how to build the initial mesh, trans-finite interpolation (TFI) with dendrites, smoothing of dendritic meshes, and a brief conclusion.

2 Data Structures

The mesh is composed of a set of logically rectangular mesh blocks with the individual blocks connected together in an unstructured fashion creating a block-structured mesh data structure. Mesh nodes for the entire problem are stored in a single master node array. The array index of a node in this master array is its $id$. The array of nodes for a mesh block stores the id’s of the nodes for the block and not the nodes themselves. Block-to-block connectivity is determined implicitly by shared nodes, not by an explicit block topology data structure.