Multi-class Network with Phase Type Service Time and Group Deletion Signal

Thu-Ha Dao-Thi\textsuperscript{1}, Jean-Michel Fourneau\textsuperscript{1}, and Minh-Anh Tran\textsuperscript{2}

\textsuperscript{1} PRiSM, Université de Versailles St-Quentin, CNRS, UniverSud Paris, Versailles, France
\textsuperscript{2} Université de Paris Est Créteil, France
{thu-ha.dao-thi,jmf}@prism.uvsq.fr, minh-anh.tran@univ-paris12.fr

Abstract. We consider networks with multiple classes of customers which receive service with a Phase type distribution. The service discipline is Last In First Out. We consider negative signal and a new type of signal: the group deletion signal. Negative signals eliminate a customer in service (if there are any) and group deletion signal delete all consecutive customers in the same class and same phase at the back-end of the buffer. We prove that the network has product form solution.

Keywords: Queue, Network, phase type, quasi-reversible, LIFO, product form.

1 Introduction

Traditional queueing networks model systems are used to represent contention among customers for a set of resources. Customers moves form server to server, waiting for service. But the customers do not interact among themselves or modify the queue or the server. G-network models overcome some of the limitations of conventional queueing network models adding signals and interactions between signals and customers. Despite this deep modification of the model, G-networks still preserve the product form property of some Markovian queueing networks. In his seminal paper \[10\], Gelenbe introduced negative customer, the first type of signal. A negative customer is never queued. A negative customer deletes a positive customer at its arrival at a backlogged queue. Positive customers are usual customers which are queued and receive service or are deleted by negative customers. Under typical assumptions for Markovian queueing networks (Poisson arrival for both types of customers, exponential service time for positive customers, Markovian routing of customers, open topology, independence) Gelenbe proved that such a network has a product form solution for its steady-state behavior. The results are more complex than Jackson’s networks. The G-networks flow equations exhibit some uncommon properties: they are neither linear as in closed queueing networks nor contracting as in Jackson queueing networks. Therefore the existence of a solution had to be proved \[11\] by new techniques, a numerical algorithm was also developed \[8\].
G-networks had been extended in many directions. First many signals were introduced and shown to lead to product form solution: triggers which redirect other customers among the queues, catastrophes which flush all the customers out of a queue \cite{12,13} and resets \cite{14}. Multiple class versions of these models have also been derived \cite{6,9,15}.

Most of the signals studied so far have a globally negative effect on the queues. Indeed, the balance of customers in the queues involved in a signal is negative (triggers, deletion, catastrophes) or negative in expectation (resets). Recently more complex interactions were introduced: change the class of the customer in service \cite{4}, change the phase of the customer in service for Phase type service distribution, synchronised arrivals in a set of queues \cite{5}. For a review, one can see these books \cite{3,16}, and many references therein.

G-networks had also motivated new important results in the theory of queues. As negative customers lead to customer deletions, the original description of quasi-reversibility does not hold anymore and new versions have been proposed. At the time being, the description proposed by Chao and his co-authors in \cite{8} looks sufficient to study queues with customers and signals. At the same time a completely different approach, based on Stochastic Process Algebra, was proposed by Harrison \cite{17,18}. The main results (CAT and RCAT theorems and their extensions \cite{11,17,18,19}) give some sufficient conditions for product form stationary distributions. Thus Harrison’s technique clearly has a different range of applications as it allows to represent component based models which are much more general than networks of queues. An interesting open question is to mix both results to obtain a quasi-reversibility characterisation directly from a SPA specification using a master-slave description of the system (like in RCAT) rather than arrivals, departure and internal transitions as proposed in \cite{3}.

Here we introduce a new type of signal which deletes several customers of the same type (same class and same phase) at the back end of a LIFO queue. Batch deletion were studied by Gelenbe in \cite{12} for single class model. To the best of our knowledge the multiclass problem was not considered until now, except the catastrophe in a PS queue studied in \cite{7} which is easier to model. Moreover, the group-deletion signal is not like the previously studied batch which was studied as we seek to delete all customers of the same type, which means that it will depend on the type of customers, while the effect of a batch or a catastrophe does not depend on class of customers. We also assume that the service time distributions are Phase type.

The following of the paper is as follows. Section 2 is devoted to the definition of quasi-reversibility as it has been generalised by Miyazawa and his co-authors to take into account signals. In section 3, we show that the queues are quasi-reversible. Finally in section 4, using quasi-reversibility we prove that the steady-state has a product form solution.

\section{Preliminaries}

In this section, we will introduce the network of quasi-reversible queues which is introduced by Chao, Miyazawa and Pinedo in \cite{3}. All the results presented in this