A Reduction of the Complexity of Inconsistencies Test in the MACBETH 2-Additive Methodology

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Abstract. MACBETH 2-additive is the generalization of the Choquet integral to the MACBETH approach, a MultiCriteria Decision Aid method. In the elicitation of a 2-additive capacity step, the inconsistencies of the preferential information, given by the Decision Maker on the set of binary alternatives, is tested by using the MOPI conditions. Since a 2-additive capacity is related to all binary alternatives, this inconsistencies checking can be more complex if the set of alternatives is very large. In this paper, we show that it is possible to limited the test of MOPI conditions to the only alternatives used in the preferential information.

Keywords: MCDA, Preference modeling, MOPI conditions, Choquet integral, MACBETH.

1 Introduction

Multiple Criteria Decision Aid (MCDA) aims at helping a decision maker (DM) in the representation of his preferences over a set of alternatives, on the basis of several criteria which are often contradictory. One possible model is the transitive decomposable one where an overall utility is determined for each option. In this category, we have the model based on Choquet integral, especially the 2-additive Choquet integral (Choquet integral w.r.t. a 2-additive) \cite{S14}. The 2-additive Choquet integral is defined w.r.t. a capacity (or nonadditive monotonic measure, or fuzzy measure), and can be viewed as a generalization of the arithmetic mean. Any interaction between two criteria can be represented and interpreted by a Choquet integral w.r.t. a 2-additive capacity, but not more complex interaction.

Usually the DM is supposed to be able to express his preference over the set of all alternatives \(X\). Because this is not feasible in most of practical situations (the cardinality of \(X\) may be very large), the DM is asked to give, using pairwise comparisons, an ordinal information (a preferential information containing only
a strict preference and an indifference relations) on a subset $X' \subseteq X$, called reference set. The set $X'$ we use in this paper is the set of binary alternatives or binary actions denoted by $\mathcal{B}$. A binary action is an (fictitious) alternative representing a prototypical situation where on a given subset of at most two criteria, the attributes reach a satisfactory level 1, while on the remaining ones, they are at a neutral level (neither satisfactory nor unsatisfactory) 0. The characterization theorem of the representation of an ordinal information by a 2-additive Choquet integral [13] is based on the MOPI property. The inconsistencies test of this condition is done on every subsets of three criteria.

We are interested in the following problem: how to reduce the complexity of this test of inconsistencies when the number of criteria is large? We propose here a simplification of the MOPI property based only on the binary alternatives related to the ordinal information.

After some basic notions given in the next section, we present in Section 3 our main result.

2 Basic Concepts

Let us denote by $N = \{1, \ldots, n\}$ a finite set of $n$ criteria and $X = X_1 \times \cdots \times X_n$ the set of actions (also called alternatives or options), where $X_1, \ldots, X_n$ represent the point of view or attributes. For all $i \in N$, the function $u_i : X_i \rightarrow \mathbb{R}$ is called a utility function. Given an element $x = (x_1, \ldots, x_n)$, we set $U(x) = (u_1(x_1), \ldots, u_n(x_n))$. For a subset $A$ of $N$ and actions $x$ and $y$, the notation $z = (x_A, y_{N-A})$ means that $z$ is defined by $z_i = x_i$ if $i \in A$, and $z_i = y_i$ otherwise.

2.1 Choquet Integral w.r.t. a 2-Additive Capacity

The Choquet integral w.r.t. a 2-additive capacity [6], called for short a 2-additive Choquet integral, is a particular case of the Choquet integral [8,9,14]. This integral generalizes the arithmetic mean and takes into account interactions between criteria. A 2-additive Choquet integral is based on a 2-additive capacity [4,8] defined below and its Möbius transform [3,7]:

**Definition 1**

1. A capacity on $N$ is a set function $\mu : 2^N \rightarrow [0, 1]$ such that:
   (a) $\mu(\emptyset) = 0$
   (b) $\mu(N) = 1$
   (c) $\forall A, B \in 2^N, [A \subseteq B \Rightarrow \mu(A) \leq \mu(B)]$ (monotonicity).

2. The Möbius transform [3] of a capacity $\mu$ on $N$ is a function $m : 2^N \rightarrow \mathbb{R}$ defined by:
   $$m(T) := \sum_{K \subseteq T} (-1)^{|T \setminus K|} \mu(K), \forall T \in 2^N.$$  (1)