Abstract. A new method is presented for determining delay-independent stability zones of the general LTI dynamics with multiple delays against parametric uncertainties. This method utilizes extended kronecker summation and unique properties of self-inversive polynomials. Self-inversive polynomials are special polynomials which exert useful tools for examination of the distribution of its zeros. A sufficient condition for delay-independent stability is presented. The main foci in this paper is a novel approach to the robustness of the time-delayed systems. A new sufficient condition for delay-independent stability is introduced. These new concepts are also demonstrated via some example case studies.

1 Introduction and the Problem Statement

We consider linear time invariant, retarded multiple time delayed systems (LTI-MTDS), general form of which is given as,

$$\dot{x}(t) = A(q)x(t) + \sum_{j=1}^{p} B_j(q)x(t - \tau_j)$$

(1)

where $x \in \mathbb{R}^n$, $A(q)$, $B_j(q)$, $j = 1 \ldots p$ are matrices in $\mathbb{R}^{n \times n}$, $q \in \mathbb{R}^r$ and the vector of time delays $\tau = (\tau_1, \tau_2, \ldots, \tau_j, \ldots, \tau_p) \in \mathbb{R}^{p+}$ the elements of which are rationally independent from each other. As a note of formalism we use boldface capital notation for vector and matrix quantities in the text. In the text, open unit disc, unit circle and outside of unit circle are referred as $\mathbb{D}$, $\mathbb{T}$, $\mathbb{S}$, respectively. Therefore, $\mathbb{D} \cup \mathbb{T} \cup \mathbb{S} = \mathbb{C}$ represents the entire complex plane. In addition to these, complex domain can be separated into to open left $\mathbb{C}_-$ and open right $\mathbb{C}_+$ half planes and $\mathbb{C}_0$ imaginary axis.

Ali Fuat Ergenc
Istanbul Technical University, Control Engineering Department, 34469, Istanbul, Turkey
e-mail: ali.ergenc@itu.edu.tr
http://web.itu.edu.tr/ergenca

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The stability of this general class of systems has been widely studied over the years \[1, 2, 3, 11, 21\]. The determination of the robustness of such systems against uncertainties in delay and other parameters (i.e., uncertain \(\tau\), \(A\) and \(B_j\)) are also investigated by many researchers \[5, 6, 7, 8, 9, 24, 27\]. The stability of the time-delayed systems can be investigated in two classes: delay-independent and delay-dependent \[1, 4, 20\]. The asymptotic stability of linear delay-differential systems independent of delay is declared as \(N-P\) hard (nondeterministic-polynomial time hard) problem \[10\]. Many studies appeared on this particular subject, on some simplified forms of the delay-independent stability problem given here \[4, 20, 23, 25, 26, 28, 29, 30, 31, 32, 33\]. In the mainstream, there are usually two approaches to the problem. First one is Lyapunov based approaches, such as Lyapunov-Krasovskii functionals which provide some sufficient delay-independent stability conditions using specific matrix inequalities. Another one is Lyapunov-Razumikhin which can be derived from the first one with a matrix operation. These methods are based on finding arbitrary positive definite matrices and it may be hard to find an appropriate one. On the other hand, they provide sufficient conditions for the stability with a conservative manner. Second approach is basically stating that the delay free system has all its characteristic roots on the left half plane and for all the delays there are no root crossings of the imaginary axis. There are several methods for the determination of the delay-independent stability criteria such as frequency (\(\omega\)) sweeping methods or matrix pencil approaches. In a recent study, a new method for delay independent stability test is also presented as a natural result of determination of the crossing frequency set \[28\].

In this study, the problem of dictating delay-independent stability criteria is transferred to assigning a certain number of the zeros in \(\mathbb{D}\) of a polynomial derived from the system equations. The key novelty introduced by the method is that it doesn’t have any restrictions on the number of the delays \((p)\) and the number of the parametric uncertainties \((r)\). It is based on unique properties of a self-inversive polynomial which represents the infinite dimensional delayed system in terms of a finite dimensional polynomial with interspersed zeros on the unit-circle.

The text is structured as follows: In section 2, preliminaries of the study are given. Section 3 states the delay-independent stability criteria for LTI system with multiple delays under parametric uncertainties. Section 4 contains illustrative example case studies.

### 2 Preliminaries

The characteristic equation of the system in (1) is

\[
CE(s, q, \tau_1, \ldots, \tau_p) = \det \left[ sI - A(q) - \sum_{j=1}^{p} B_j(q)e^{-\tau_j s} \right] \\
= A_0(s, q) + A_{p+1}(s, q, \tau_1, \ldots, \tau_p) + \sum_{j=1}^{p} e^{-\eta_j \tau_j s} A_j(s, q, \tau_1, \ldots, \tau_{j-1}, \tau_{j+1}, \ldots, \tau_p) = 0
\] (2)