Anti-diffusion Interface Sharpening Technique for Two-Phase Compressible Flow Simulations

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1 Introduction

Shock waves in two-phase compressible flows are a fundamental topic in science and engineering. To better understand instability phenomena that are important for the evolution of such flows, basic configurations such as shock-bubble interactions in two-phase compressible flows are considered to investigate the Richtmyer-Meshkov instability and Rayleigh-Taylor instability. Flows of this type are present in many engineering applications including supersonic mixing and combustion systems and extra-corporeal shock-wave lithotripsy.

Numerical models of two-phase compressible flows play a significant role in the study of the topic as they provide the access to flow regimes and quantities which cannot be studied and obtained analytically and experimentally. The main numerical methods for two-phase compressible flow simulations are the level-set methods [1], the volume-of-fluid (VOF) methods [2], and front-tracking methods [12].

The VOF volume-capturing method possesses the advantage of exact conservation properties, but suffers from numerical diffusion which causes two-fluid interfaces to smear. Specific numerical schemes to suppress or counter-act the numerical diffusion, and to maintain the interface sharpness in the course of simulations are thus desirable for VOF methods. Previous works include the interface compression technique by [9], and the anti-diffusive numerical scheme based on a limited downwind strategy [5].

In this paper, we propose an interface sharpening technique for two-phase compressible flow simulations. The idea is to solve an anti-diffusion equation for counter-acting the numerical diffusion. The technique has been developed and verified for two-phase incompressible flow simulations [10]. It is the objective of this paper to present the key concepts and numerical results of the application of the technique to two-phase compressible flow simulations.

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2 Governing Equations

We consider the Euler equations assuming a single velocity and pressure equilibrium. The two phases are represented by the volume fractions, where the formulation of the volume-fraction equations of [2] is adopted. The volume-fraction equation formulation has been extensively studied by [6] for simulations with ideal-gas equation of state (EOS) and Mie-Grüneisen EOS, and serves as the governing equations for a computational study for shock-bubble interactions by [7]. With two mass conservation equations, one momentum conservation equation and one energy conservation equation a six-equation model is obtained as follows:

\[ \frac{\partial \alpha}{\partial t} + \nabla \cdot (\alpha \mathbf{u}) = \alpha \frac{K_S}{K_\alpha} \nabla \cdot \mathbf{u}, \quad (1) \]

\[ \frac{\partial \beta}{\partial t} + \nabla \cdot (\beta \mathbf{u}) = \beta \frac{K_S}{K_\beta} \nabla \cdot \mathbf{u}, \quad (2) \]

\[ \frac{\partial \alpha \rho^\alpha}{\partial t} + \nabla \cdot (\alpha \rho^\alpha \mathbf{u}) = 0, \quad (3) \]

\[ \frac{\partial \beta \rho^\beta}{\partial t} + \nabla \cdot (\beta \rho^\beta \mathbf{u}) = 0, \quad (4) \]

\[ \frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) + \nabla p = 0, \quad (5) \]

\[ \frac{\partial E}{\partial t} + \nabla \cdot (E + p) \mathbf{u} = 0, \quad (6) \]

where \( \alpha \) and \( \beta \) are the volume fractions of the two phases respectively and \( \alpha + \beta = 1 \), \( t \) is the time, \( \mathbf{u} \) is the velocity, \( K_S \) is the mixture bulk modulus, \( K_\alpha \) and \( K_\beta \) are the phase bulk moduli, \( \rho^\alpha \) and \( \rho^\beta \) are the phase densities, \( p \) is the pressure and \( E \) is the total energy.

3 Numerical Methods

3.1 Riemann Solver

The HLL Riemann solver [13] is adopted for calculating the numerical flux at cell face, \( \mathbf{F}_{HLL} \),

\[ \mathbf{F}_{HLL} = \begin{cases} \mathbf{F}_L & \text{if } 0 \leq S_L, \\ \frac{S_R \mathbf{F}_L - S_L \mathbf{F}_R + S_L S_R (U_R - U_L)}{S_R - S_L} & \text{if } S_L \leq 0 \leq S_R, \\ \mathbf{F}_R & \text{if } 0 \geq S_R, \end{cases} \quad (7) \]