1 Introduction

In the case of breakdown waves in a long discharge tube, near the electrode where the potential gradient in the gas is greatest, small quantity of gas is ionized. Analysis of the spectrum of radiation emitted from electric breakdown of a gas reveals no Doppler shift, indicating that the ions have negligible motion. The large difference in mobilities of positive ions and electrons causes establishment of a space charge and consequently a space charge field. The electric field accelerates the free electrons until they acquire enough of energy for collisional ionization of the gas. Since the ionized gas is a conductor and it can not hold internal electric filed, the electric field which has its maximum value at the interface between the ionized gas and the neutral gas has to reduce to a negligible value at the trailing edge of the wave.

Breakdown waves for which the electric field force on electrons causes the average drift velocity of the electrons to be away from the wave front are referred to as antiforce waves. In the case of antiforce waves, the electron fluid pressure is considered to be large enough to provide the driving force and cause the propagation of the wave down the tube with observed velocities. This implies that despite the net electron drift velocity away from the wave front, the electron temperature must be large enough to sustain the wave motion. The wave is composed of two distinct regions. Following the shock front is a thin dynamical region in which the electric field starting from its maximum value at the shock front reduces to a negligible value at the trailing edge of the wave and the electrons slow down to speeds comparable to those of ions and heavy particles. This region will be referred to as the sheath region. Following the sheath region there is a relatively thicker thermal region in which the electron gas cools down by further ionization of the wave. This region will be referred to as the quasi-neutral region.
2 Model

In our investigation of breakdown waves, we will employ the set of differential equations for the structure and state of the electron gas behind the shock front developed by Fowler et al.\[1\]. The basic equations for analyzing breakdown waves for the sheath region are the equations of conservation of mass, momentum, and energy coupled with Poisson’s equation, and they respectively are

\begin{equation}
\frac{d(nv)}{dx} = n\beta, \tag{1}
\end{equation}

\begin{equation}
\frac{d}{dx}[mnv(v - V) + nkT_e] = -enE - Kmn(v - V), \tag{2}
\end{equation}

\begin{equation}
\frac{d}{dx}[mnv(v - V)^2 + nkT_e(5v - 2V) + 2env\phi - \frac{5nk^2T_e}{mK} \frac{dT_e}{dx}] = -3\left(\frac{m}{M}\right)nkT_e - \left(\frac{m}{M}\right)Kmn(v - V)^2, \tag{3}
\end{equation}

\begin{equation}
\frac{dE}{dx} = \frac{e}{\varepsilon_0}(N_i - n). \tag{5}
\end{equation}

where \(n, v, T_e, e\) and \(m\) represent the electron number density, velocity, temperature, charge, and mass, respectively, and \(M, E, E_0, V, k, K, x, \beta\) and \(\phi\) represent the neutral particle mass, electric field within the sheath region, electric field at the wave front, wave velocity, Boltzmann’s constant, elastic collision frequency, position within the sheath region, ionization frequency and ionization potential of the gas. The ion number density within the sheath region is represented by \(N_i\).

For breakdown waves moving into a non-ionized medium, the net current ahead of the wave will be zero. This condition, \(e(N_iV - nv) = 0\), is known as the zero current condition. Using the zero current condition in Eq.(5) reduces it to

\begin{equation}
\frac{dE}{dx} = \frac{e}{\varepsilon_0}(v - V). \tag{6}
\end{equation}

In solving the antiforce case problem, we will use the set of non-dimensional variables and the set of fluid-dynamical equations derived by Hemmati\[5\]. Hemmati’s set of non-dimensional variables are

\begin{align*}
\eta & = \frac{E}{E_0}, \nu = \left(\frac{2e\phi}{\varepsilon_0 E_0^2}\right)n, \psi = \frac{v}{V}, \theta = \frac{T_e k}{2e\phi}, \xi = -\frac{eE_0x}{mV^2},
\end{align*}

\begin{align*}
\alpha & = \frac{2e\phi}{mV^2}, \kappa = -\frac{mV}{eE_0} K, \mu = \frac{\beta}{K}, \omega = \frac{2m}{M},
\end{align*}

in which \(\eta, \nu, \psi, \theta, \mu\) and \(\xi\) represent the dimensionless net electric field of the applied field plus the space charge field, electron number density, electron velocity, electron gas temperature, ionization rate, and position within the sheath region, while \(\alpha\) and \(\kappa\) represent wave parameters.