Chirp Watermark Detection Based on Fractional Auto-correlation Statistic

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Abstract. To overcome the shortage of the existing detector algorithm when the focus order of Chirp watermark near ±1, a novel blind detection method based on fractional auto-correlation statistic has been proposed. The theory and methods of implementation have been discussed. Simulation results show that the algorithm is better robust to common attack.

Keywords: Fractional Fourier transform, digital watermarking, Chirp signal, fractional auto-correlation.

1 Introduction

The prompt booming of internet has brought abundant opportunities to commerce, scientific research and entertainment by the various services and means of electronic publishing and printing, electronic advertisement, digital library, internet videos and audios, e-business and etc. The digital multi-media products could be easily copied and extensively spread. Digital watermark technique provides a solution to this dilemma, which could be used for copy control and prohibiting unauthorized copy, and protects the copyright in transmission and application [1].

So far, several methods have been proposed to achieve this purpose. The digital watermark embedding and detecting algorithms mainly perform in the time/spatial domain or transform domain. Fractional Fourier transform shows the best focus property [2,3] on a given Chirp signal in a specific fractional order. The Chirp signal, with linear frequency and geometric symmetry and the relatively larger time-bandwidth product, represents relatively better robustness to common Stationary filtering and affine transform. Therefore two dimensional Chirp signal has been used as digital watermark embedded into image. The proposal of multi-component Chirp watermark algorithm borrowed the theory in [4] as to the combination of spatial domain/frequency domain technique, which directly embeds the Chirp watermark signal into the spatial domain of the image and the detects by Radon-Wigner transform (RWT) [3].
Consider a watermarked image \( I'(x, y) \) with the same size of \( N \times N \) as the original image \( I(x, y) \): \( I'(x, y) = I(x, y) + \lambda C(x, y) \), where \( \lambda \) is the watermark amplitude or strength, and a compromise between the robustness of the watermarking algorithm and the invisibility of the watermark. The 2-D multi-component and real-value chirp signal, used here as a watermark, has the following form:

\[
C(x, y) = \sum_{n=1}^{K} A_n \cos \left( n \mu_{ \alpha x } x^2 + 2 \pi \mu_{ \alpha y } x + n \mu_{ \beta y } y^2 + 2 \pi \mu_{ \beta y } y \right)
\]

(1)

Where \( K \) is the number of components and \( A_n \) is the amplitude of each component.

The RWT is equal to the squared magnitude of the FRFT of the signal [8], therefore a series of Chirp watermark detection methods based on fractional Fourier transform has been proposed recent years [5-7]. Although those detection methods could detect the Chirp watermark very well by virtue of concentration on Chirp signal, there are some limitations. Because the 2D fractional Fourier transform is the continuous time-frequency plane between spatial domain and frequency domain, when the transform order is approaching 1, the energy of the image is also in a process of converging to the center. Once the corresponding focus order of each Chirp watermark component approaches to \( \pm 1 \), it would be inevitably influenced by the converging effect of image energy, which easily leads to difficulty of searching the Chirp concentration peak or leads to False alarm. Thus the watermark detection method proposed in [5-7] applies only to the situations in which the Chirp watermark focuses on a smaller FRFT order and the key space is restricted. For that reason, a novel blind detection method to 2-D multi-component Chirp watermark has been proposed in this paper.

2 The Principles of Fractional Auto-Correlation Statistic and Chirp Signal Detection

As the ambiguity function (AF) of Chirp signal is a radial line through the origin of the ambiguity plane [2,3] and the slope \( m \) corresponds to the Chirp rate of the signal, integrating magnitude AF along a line with angle \( \phi \) going through the origin equals to integrating fractional auto-correlation with the angle \( \phi \) of the received signal, thus the following can be derived:

\[
L(\phi) = \int \left| (s \otimes_{\rho} s) (\rho) \right| d\rho
\]

(2)

Let Chirp rate perform as variable, the fractional auto-correlation statistic can be further rewritten as:

\[
L(m) = \int \left| (s \otimes_{\text{arctan}(m)} s) (\rho) \right| d\rho
\]

(3)

The fractional auto-correlation of signals is given as:

\[
(s \otimes_{\rho} s)(\rho) = F^{-\pi/2} \left\{ S_{\pi/2+\rho}^2 (u) \right\} (\rho)
\]

(4)