# Chapter 13

Small Strain Nonlinearity

**TABLE OF CONTENTS**

13.1 Introduction ........................................................................................................... 156

13.2 Non-linear models of the small-strain behaviour ............................................... 157

13.2.1 Normalized pre-yielding behaviour ................................................................. 157

13.2.2 The hyperbolic function .................................................................................... 159

13.2.3 The Ramberg-Osgood function ......................................................................... 160

13.2.4 The logarithmic function .................................................................................. 160

13.3 Modelling irreversible small strain behaviour ...................................................... 161

13.3.1 Masing rules .................................................................................................... 161

13.3.2 Modelling cyclic and dynamic small strain behaviour ..................................... 162

13.3.3 Comparison between the non-linear models ..................................................... 163

13.3.4 The proper theoretical framework .................................................................... 164

13.4 Thermomechanics of the small strain non-linearity ............................................. 164

13.4.1 Reversible non-linear behaviour and Thermomechanics .................................. 164

13.4.2 Irreversible non-linear behaviour and Thermomechanics .............................. 166
Chapter 13
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13.1 Introduction
In the previous chapter, in the models with pressure dependent stiffness, the shear modulus $G$ was either constant or increased with the mean effective stress. These models used to describe the pre-yielding deviatoric behaviour of soils in standard triaxial compression tests reasonably well. The problem was that the corresponding numerical models considerably over-predicted displacements in many boundary value problems. In late 1980s this discrepancy lead researchers at Imperial College, London (e.g., Burland, 1989), to an idea to conduct laboratory tests with strains measured locally on the sample, as opposed to the external strain measurements in standard triaxial tests (Figure 13.1a). When plotted in a wide strain range (Figure 13.1b), the deviatoric stress-strain curves of the two tests do not differ that much. The curve for the local strain test (dashed line) goes slightly higher, but this does not affect the pre-yielding secant shear modulus $G$ significantly. A different picture is observed when we zoom (Figure 13.1c) into the area of very small strains (up to 0.01-0.1%). While the externally measured stress-strain test curve (solid line) is almost linear, the locally measured stress-strain curve proves to be highly non-linear, with the initial tangent shear modulus $G_0$ almost an order of magnitude higher than the pre-yielding secant shear modulus $G$. Another important discovery was that even at the very small strains the stress-strain behaviour is not entirely reversible, i.e. it exhibits some very small permanent strains in a closed loading cycle (Figure 13.1c).

![Figure 13.1 External vs. local deformation measurements in a triaxial test: (a) schematic setup; (b) deviatoric stress-strain curves; (c) zoom into small strains.](image)